VOLUME I

Chapter IV

Spectrophotometry

Kozyriev

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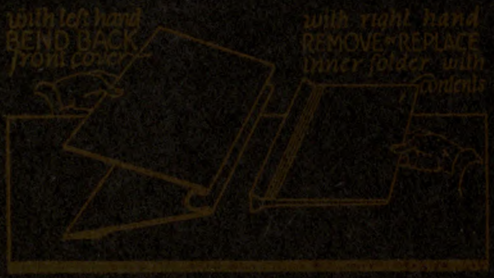

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Chapter IV

Spectrophotometry

by N. A. Kozyrev

1. Direct Spectrophotometry

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Chapter IV

Spectrophotometry

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1. Direct Spectrophotometry

1. Basic Principles of Direct Spectrophotometry. Depending on the purpose of a spectrophotometric investigation, various instruments are used for the decomposition of light. Usually two basic types of spectroscopes, the slit and the slitless spectroscopes, are used. The slit spectroscopes yield sharp spectra of large dispersion with spectral lines that are washed out very little. Therefore it is best to use these spectroscopes or spectrographs for the study of line contours. In general they are of little use for the measurement of the distribution of energy in the continuous stellar spectra because the slit cuts out a part of the stellar image. However, in the investigation of the continuous spectrum of surfaces (the solar surface or details on the solar disk, for instance) these spectroscopes and spectrographs are advantageous because their large dispersion enables us to measure the intensity of a truly continuous spectrum when we select portions which are free of lines. As we know already from Chapter 2, slit spectroscopes and spectrographs are of two kinds: those with prisms and those with diffraction gratings. The spectroscopes and spectrographs with a diffraction grating may serve for spectrophotometric purposes only when applied to very bright objects because of the great loss of light in these instruments. As the slit spectroscopes with prisms cause also a great loss of light (for spectroscopes of three prisms the loss is about 97-98 o/o), slitless spectroscopes, chiefly the objective prisms are used for the study of spectra of faint stars. The objective prism has its greatest application in the study of the continuous spectra of stars.

In spectrophotometric investigations of stars it is usually necessary to widen their spectra artificially. In visual observations it is done by means of cylindrical lens, in photographic observations it is done by shifting the stellar image. Let us assume now that, having obtained the spectrum, it is necessary to measure the intensity of the radiation at a certain interval of wave lengths . The ideal problem of spectrophotometric measurements is the obtaining of this quantity in energy units for each wave length. Such absolute measurements may be made by means of transfer of the radiant energy into heat energy which is measured by a bolometer, radiometer, or thermoelement. These instruments may be applied only to spectra of very bright objects. Besides, in order to obtain the intensities in absolute units it is necessary to keep track of the loss of energy as the light passes through the terrestrial atmosphere and the measuring apparatus (a very difficult and complex process.) Fortunately, relative measurements are sufficient for the solution of numerous astrophysical problems. The following principle, applicable to all photometry, is the basis of all spectrophotometric comparisons. Namely, for the determination of brightness of the spectrum of a certain object with respect to the spectrum of the given object we equalize the brightness of the compared portions of the spectrum by means of changing the brightness of one of the objects as many times as the number measured. Or, introducing an auxiliary object, its brightness is changed until it coincides with the brightness of the other two sources. Thus, photometric scales are obtained. Photometric comparisons may be made either directly with the eye or by means of a photographic plate.

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The photographic plate has many advantages as compared with the eye. The most important of them are as follows: the photographic plate is sensitive to the rays of a considerably larger portion of the spectrum than the eye (the possibility of studying the ultra-violet and the infra-red regions of the spectrum); next, the plate may be applied to faint objects which are inaccessible to the eye; and, finally, the development of the plates takes place under laboratory conditions which are more convenient for measurements than the places of observation. Because of these advantages, photographic spectrophotometry is almost exclusively used in astronomical practice at the present time.

In spectrophotometric comparisons as well as in all measurements we should try as much as possible to obtain more differential comparisons by having the measured object and the object of comparison under as much the same conditions as possible. It is very important to make the comparisons for portions of the spectrum of the same wave length. This principle was not maintained in the early spectrophotometers. Visual comparisons of intensities of light of different colors were made. Precisely in this way Fraunhofer had first determined in 1814 the distribution of energy in the continuous solar spectrum. The degree of error in such measurements is very large. In the study of line contours the intensities are expressed in units of radiation of the continuous spectrum of the neighboring regions. Thus, in this problem the condition of the same wave-length is easily realized. In obtaining the curves $I(\lambda)$ for the continuous spectrum it is necessary to express the I_λ in units of the intensity of a certain definite wave length. Evidently, in order to maintain the condition, mentioned above, it is necessary to make the comparison of the intensity of the spectrum of the object with the spectrum of a certain terrestrial source, for instance, of a known distribution of energy. Such a source may be an absolutely black body for which the $I(\lambda)$ is determined according to Planck's formula. Thus, we may find the distribution of energy in the spectrum of a certain star. The spectra of other stars may be referred to this spectrum. In the latter case a large differentiation is obtained; the errors of the instrument may be entirely excluded and the correction for the atmospheric transparency will also enter only differentially. It is precisely this problem of comparing the spectra of the celestial objects that the observer-astronomer must deal with. Knowing the $I(\lambda)$ for a certain star in some arbitrary units, we may also obtain the absolute values of these intensities from the integrated stellar brightness in absolute units. Indeed, as the integrated brightness within a certain range of wave lengths in units of energy is equal to $K\int I(\lambda)d\lambda$ we may comput the constant K - the factor which transforms the relative intensity into the absolute intensity. The measurements of the integrated intensities in units of energy are simpler than in the case of monochromatic radiation. Therefore the absolute spectrophotometric evaluations are usually made in this way, i.e. on the basis of the absolute measurements of the integrated brightnesses.

§2. The Selective Absorption of Light by the Terrestrial Atmosphere. The stellar light passing through the terrestrial atmosphere is partially absorbed. This absorption is selective, i.e. different for various wave lengths. Therefore the observed distribution of energy in the stellar spectrum differs from the real distribution. The far ultra-violet regions of the spectrum for $\lambda < 290\mu\mu$ are almost entirely absorbed by the atmosphere. This absorption is caused by bands of ozone adjoining closely to each other. For the

far infrared rays $\lambda > 4\mu$, the atmosphere is also not sufficiently transparent. This is explained by the fact that in the infrared region most of the molecules yield intense bands of absorption. In the terrestrial atmosphere such an absorption is caused chiefly by water vapor and carbon dioxide.

In the visual and photographic regions of the spectrum there are also lines of absorption of the terrestrial atmosphere -- the so called telluric lines, but here they do not yield continuous bands, and it is quite possible to select portions of the spectrum which are free from the telluric lines. The continuous absorption in this region of the spectrum is caused chiefly by the scattering of light by the molecules of the air; in the far ultraviolet region of the spectrum the air yields very intense absorption lines whose wings reach into the visible part of the spectrum. This absorption changes with the wavelength according to Rayleigh's law, i.e. in proportion to λ^{-4}.

Let us see now how we may obtain a record of this selective absorption of the atmosphere. Aside from a complete liberation of the observed intensities from atmospheric absorption, "reductions to the zenith" are sometimes made as an intermediary stage. Let us consider both of these cases of reduction. Let us denote the observed brightness of a star at a zenith distance z by $I_z(\lambda)$ and its true brightness by I   Then.

$$I_z\lambda = I(\lambda)\,e^{-k(\lambda)\int \rho\, ds} \quad . \qquad (1)$$

where k (λ) is the coefficient of absorption referred to the unit of mass and it is assumed to be constant for the entire atmosphere,   is the density, and ds is the element of the path. $\int \rho ds$ represents the entire mass of the air through which the ray of light passes. Let us denote it by M_z.

Then equation (1) and the similar equation for the zenith are written as follows:

$$I_z(\lambda) = I(\lambda)\,e^{-k(\lambda)M_z},$$
$$I_0(\lambda) = I(\lambda)\,e^{-k(\lambda)M_0}\,. \qquad (a)$$

Let us call the ratio

$$\frac{I_0(\lambda)}{I(\lambda)} = P(\lambda) \qquad (b)$$

the coefficient of transparency (the coefficient of transmission) of the atmosphere for a given   Then

$$\frac{I_z(\lambda)}{I_0(\lambda)} = P(\lambda)^{F(z)-1} \qquad (2)$$

$$\frac{I_z(\lambda)}{I_\lambda} = P(\lambda)^{F(z)} \qquad (3)$$

where

$$F(z) = \frac{M_z}{M_0}\,. \qquad (c)$$

If we assume the layers of the terrestrial atmosphere to be parallel planes, then

$$F(Z) = \sec z\,.$$

The computation of function $F(z)$ taking into consideration of the curvature of layers, the refraction, and the changes of the temperature as the altitude increases was made by Bemporad (see Chapter III, §4). His tables of the function $F(z)$ are used at the present time. The deviations of Bemporad's numbers from sec z are noticeable practically only at zenith distances which are greater than 70°.

Formula (2) enables us to make a reduction to the zenith. Formula (3) enables us to obtain the true distribution of energy $I(\lambda)$ in the stellar spectrum. $P(\lambda)$ remains unknown in these formulae; its values may be obtained by observing the same star at various zenith distances and determining $I_z(\lambda)$. Then we may easily compute $P(\lambda)$ by the formulae (2) and (3). The values of $P(\lambda)$ depend, evidently, on the altitude of the place of observation above sea level and on metearological conditions. The determination of the mean values of $P(\lambda)$ was carefully made by Müller and Kron visually for the Teneriff Observatory, by Wilsing for Potsdam, and by Abbot for Washington and Mount Wilson (by bolometric measurements). As an illustration let us give here the values of $P(\lambda)$ obtained by Abbot for Mount Wilson:

0.325μ	0.375μ	0.420μ	0.450μ	0.500μ	0.600μ	0.800μ	1.00μ	1.300μ	2.000μ
0.550	0.662	0.764	0.800	0.850	0.890	0.964	0.973	0.972	0.957

It appears from this table that the function $P(\lambda)$ increases as λ increases which is to be expected according to Rayleigh's law. The decrease of the transparency in the infra-red rays is caused by the absorption of light by water vapor. The water vapor in this region of the spectrum yield three very intense bands of absorption:

Φ at $\lambda = 1130\ \mu\mu$, Ψ at $\lambda = 1470\ \mu\mu$ and Ω at $\lambda = 1890\ \mu\mu$.

Formula (3) may vary in several ways depending on the conditions of a given definite problem. Thus, for instance, Wilsing, Scheiner, and Munch, in their classical visual investigations of the distribution of energy in the continuous spectrum of stars, used in the computation the following formula easily obtained from (2) and (3):

$$\lg I(\lambda) = \lg I_z(\lambda) + \frac{\lg P(\lambda)}{\lg P} \lg E_z - \lg P(\lambda). \tag{4}$$

where P is the mean coefficient of transparency, i.e. the ratio of visual brightness of a star observed at the zenith to the brightness of the same star outside the atmosphere, and E_x is the ratio of the visual brightness of the star at the zenith to the brightnes of the star at a zenith distance z. The values of the $\lg E_z$ may be taken from Müller's tables given in Chapter III. In the study of the distribution of energy $I_{,}(\lambda)$ in the spectrum of a star which is at a zenith distance z by

comparing it with the spectrum of another star of a known I$_2$ (λ) a zenith distance Z$_2$, it is sufficient to introduce a correction only for the differential absorption of the terrestrial atmosphere. From formula (3) we have:

$$\lg \frac{I_1(\lambda)}{I_2(\lambda)} = \lg \frac{I_{z_1}(\lambda)}{I_{z_2}(\lambda)} + [F(z_2) - F(z_1)] \lg P(\lambda)$$

In those cases, when it is sufficient to find the distribution of the energy in the spectrum of the star, expressed in arbitrary units, the following formula may be used.

$$\lg \frac{I_1(\lambda)}{I_2(\lambda)} = \lg \frac{I_{z_1}(\lambda)}{I_{z_1}(\lambda)} + F(z_2) - F(z_1) \lg \frac{P(\lambda)}{P}$$

where P is the coefficient of transparency for certain arbitrary wavelengths. This last formula contains only the deviation P (λ) from a certain mean value, and not its absolute value. Usually, during photometric comparisons of the stellar spectra, the stars are compared, as far as the conditions of the problem allow, at equal zenith distances. Then the correction for the atmospheric transparency is entirely eliminated, if during the interim between the making of the measurement of the two stars, the coefficient of transmission has not changed its value (the constancy of the ratio is important and not the absolute value of the coefficient of transmission.

3 Visual Spectrophotometry.

Visual spectrophotometric comparisons can be made only within the comparatively narrow limits of the spectrum from λ =400μμ to λ =720μμ. For wavelengths $<$ 400μμ and $>$ 720μμ the normal eye reacts to the light impulses very little if at all. For λ =555μ the sensitivity of the eye is at a maximum. In order to have an idea of the changes of the sensitivity of the eye parallel to the changes of the wavelength, let us study a certain function V(λ), that is called the visibility function.

$$V(\lambda) = \frac{I(\lambda_{max})}{I(\lambda_{min})} \qquad (5)$$

Here I(λ) are the intensities of the radiation of different wavelengths that excite the eye reflexes in the same degree. I(λmax) represents the value of I(λ) with corresponding to the maximum sensitivity. The function I(λ) varies within quite wide limits for different eyes. Table I gives the values of the function of visibility for a certain average eye, according the data prepared by Henning (1919).

Table 1

λ B μμ	V (λ)	B μμ	V (λ)	B μμ	V (λ)	B μμ	V (λ)
440	0.053	520	0.775	580	0.843	640	0.191
464	0.184	530	0.888	590	0.739	650	0.101
480	0.281	540	0.968	600	0.624	660	0.051
490	0.369	550	0.994	610	0.503	680	0.014
500	0.495	560	0.984	620	0.395		
510	0.638	570	0.932	630	0.285		

Table 1 is prepared for the cases of light perceptions of "normal intensities". With low light perceptions or in so called "twilight", the visibility curve changes it appearance, with the maximum of the sensitivity of the eye shifted toward the blue part of the spectrum - $\lambda max \cong 0.520 \mu\mu$ and the eye ceases to react to the red color almost completely. This by the way, furnishes an explanation of the bluish color of moonlight, as compared to suhlight. In fig. 58 the curve on the right represents the visibility function for normal light, while the one on the left, that for twilight. This effect of the higher sensitivity of the eye to the blue rays in twilight has been known for a long time and is called the Purkinje effect. Physiologically the Purkinje effect is explained by the fact, that in bright light the principal part in creating the sensation of light is played by one photo-sensitive element of the eye (the "cones"), and in twilight, by the other (the "rods"). Very thorough studies by Alisso and Dangeon have shown, that the visibility curve changes also in normal light, along with the variations of the intensity of the light perception or, which is the same, with the variations of the intensity of radiation. The appellation of the "Purkinje Effect" is now generally applied to the variation of the function $V(\lambda)$ with the variation of the intensity of radiation.

According to formula (5) the existence of The Purkinje effect means, that with the sources of radiation of different intensities the ratio of the intensities, required for equalization of the light effects from two parts of the spectrum, should be, generally speaking, different. For an illustration of the Purkinje effect we include here Table 2, compiled according to Dangeon.

Table 2

m	Δm = m (Blue) - - m (Red)
- 1.1	+ 2.21
0.0	29
+ 0.7	43
+ 1.5	37
+ 2.2	39
+ 3.0	35
+ 3.7	39
+ 4.5	28
+ 5.2	27

In this table m is the stellar magnitude of the source of radiation, and $\Delta m = 2.512 \lg \frac{I \text{ red}}{I \text{ blue}}$. If the Purkinje effect was not present, Δm would remain constant.

Owing to the Purkinje effect, it is impossible to make exact spectrophotometric measurements, based on the equalization of the light effect from

different color bands. Table 2 shows, that such comparisons may be in error by quantities of the order of 0.2 stellar magnitude. Even aside from the Purkinje effect the problem of a comparison of the radiations of different colors, presents big difficulties, purely of a practical nature. It is true that at the present time these difficulties are overcome to a large degree by the use of the blink photometer. This photometer is based on the principle, that with the increased speed of changing the field of different colors (blinking), the ability of the eye to distinguish the colors of the fields is lost long before the ability to distinguish the brilliance of these fields. The result is that it is possible to compare the relative brilliance of the fields, whose colors are no longer distinguishable in this way, now, the visibility curve of the eye is determined.

During a comparison of the bands of the spectrum of a given λ, all these difficulties disappear, and it becomes possible, without knowing anything about the peculiarities of our eyes, to make accurate photometric comparisons. This is exactly the path along which astronomical visual spectrophotometry was developed. The spectrometers that were most widely employed in astronomical investigations were those of Glan-Vogel and Crovat.

In the spectrophotometer of Glan-Vogel the photometric unit is placed between the objective of the collimator and prism of the spectroscope. It consists of a doubly refracting quartz crystal (Wollaston's Prism) and a nicol prism. The ray of light, after passing through the quartz crystal separates into two rays, whose polarization planes are perpendicular. Their relative brilliance may be changed by the rotation of the nicol prism. The slit of the spectroscope is divided lengthwise by a narrow band into two parts; thus the observer sees, through the telescope, four spectra. The two outer spectro are obsecured by a diaphragm, contained in the telescope. The band, obscuring the slit is selected of such a width, that the two middle spectra touch each other. In as much as the refraction of the quartz is different for various wavelengths, it is evident, that the contact of the two spectra, can be achieved only for some part of the spectra, corresponding to a certain wavelength. The precision of the contact can be corrected by moving the collimator and, changing, correspondingly the position of the eyepiece of the telescope. The telescope contains a diaphragm with a slit, perpendicular to the spectrum and enabling us to cut out as narrow a band of the spectrum, as may be desired.

In stellar photometry, the ray of light from the star, after being sperad by a cylindrical lens, is passed through one half of the slit of the spectroscope. Over the other half of the slit is placed a totally reflecting prism which sends the light from some terrestrial source of light, used for comparison, also through the spectroscope. By measuring the angle of rotation of the Nicol Prism that is necessary to equalize the intensity of the spectra of the star and of the comparison source, the relative intensity of two given parts of the spectra can be determined. Of course the intensities will be computed according to the law of the squares of tangent.

The objectionable feature of the photometer of Glan-Vogel is the fact,

that the light from the star passes through the whole photometric system and is greatly weakened. This fault is avoided in the apparatus of Crovat.

In the spectrophotometer of Crovat, the photometric unit is placed outside the spectroscope. It consists of two nicol prisms, that can be rotated relatively to each other. The light of the star, passes through one half of the slit and then, only through an ordinary spectroscope. The other half of the slit is fitted with a small totally reflecting prism, which sends into the slit the light from the comparison source, that passed, previously, through the Nicol Prisms. The two spectra can be brought into contact along their entire length. Whenever the equality of the intensity of two spectral bands of the same wavelength is achieved, the ratio of the intensities of the spectra of star and of the terrestrial source, for that given wavelength, is found as the square of the sine of the angle of rotation of the Nicol Prism.

The first spectrophotometric measurements of the stars were made at the end of the seventies of the last century with the Glan-Vogel Spectrophotometer, the observations being performed by Voget at the Potsdam Observatory. A kerosene lamp was used as a comparison source, as the observations have shown, that its intensity changed very little with the time. The comparisons were made for seven parts of the spectrum in the interval of the wavelengths from 440μμ to 640μμ. Owing to the heavy loss of light in the apparatus, it was possible to observe only the brightest stars, not under those of the first stellar magnitude. The observers met with many difficulties. Owing to the vibration of the image, the spectrum of the star had a different appearance from that of the kerosene lamp. Also the observations were hindered by a faulty driving clock. Owing to all of these reasons, the accuracy attained was very low. Vogel estimated the error of observation as 20 - 30°/o.

These observations present now only an historical interest. Since 1905 Wilsing and Seheiner renewed the spectrophotometric studies at the Patsdam Observatory, and after 1911 they were joined by Münch. These observations have embraced 199 stars and are considered now to be of first class precision. A Crovat spectrophotometer, mounted on the 80-cm refractor of that observatory was used in the observations. The image of the star received at the slit was lengthened by a cylindrical lens. In order to avoid the errors due to chromatic aberration of the refractor and the dispersion of light by the terrestrial atmosphere, the slit of the spectroscope was widened to 1 mm in order to admit the complete chromatic image of the star. In this case the absorption lines were greatly blurred, almost to the point of complete disappearance. With an idea of avoiding the errors of determination of the intensity in the continuous part of the spectrum, due to the blurred absorption lines, the parts of the spectra selected for measurement were as free from the absorption lines, as possible. The selected bands were distributed more or less regulary in the interval between the wavelengths of 451μμ to 642μμ. A carbon filament incandescent light was used as a comparison light, and it was checked in the laboratory against the spectrum of a black body. During the observations the constancy of the current was maintained by anaddition or diminution of additional resistance. Thus, the distribution of the energy in the spectrum of

of the comparison source was well known. In order to make the spectrum of the lamp resemble that of the star, as much as possible, a diaphragm with a slit was introduced between the totally reflecting prism and the nicol prism. By changing the width of this slit it was possible to adjust the appearance of the comparison spectrum to that of the star.

A study of the selective absorption of light by the system employed to project the image of the star on the slit of the spectroscope was made for computations of the actual distribution of the energy in the spectrum of the star. This correction for the selection absorption was determined by means of a comparison of the spectrum of a brightly lit matt surface, received directly and through the system. The mean error was found to equal ± 0.024 in logarithms of ratios of the intensities, or 5°/o. Such precision is very high even for contemporary spectrophotometry. Therefore in the comparative spectrophotometric studies, the stars investigated by Scheiner, Wilsing and Munch are often used as reference stars.

Although the described measurements were of the highest accuracy, until the present day there was not a single attempt made at further visual spectrophotometric studies of the stars. The reason lies in the extreme difficulties of the visual measurements. Besides,tthe application of the photographic plate to spec rophotometry, not only introduced technical improvements over the visual method of observation, but opened new possibilities for spectrophotometric investigations. We have in view the possibility of a simultaneous measurements in the different parts of the spectrum, the study of the spectra of faint stars, the measurements in those parts of the spectra invisible to the eye, and finally the possibility of the most detailed study of different parts of the spectrum through the computing of the contours of the lines. Before describing the methods of photo-spectrophotometry, let us review the specrtophotometric peculiarities of the photographic plates.

§4 - The Nature of Photographic Plates and the Fundamental Principles of Photographic Spectrophotomerty.

The light, falling on the plate, causes blackening that becomes visible after development. In as much as the intensity of the blackening S depends on the intensity of radiation that fell upon the plate, photometric studies are based on the measurement of this degree of darkening. It is entirely immaterial, what unit or what scale is used to measure the darkening, as the measurement S in photometric studies is only an intermediate stage in the determination of I. For example, it is possible to let a ray of light pass through the image on the plate and to compare its intensity with that of a ray that passed through the photometric wedge, and thus express the darkening of the image in the terms of the graduations of this wedge. Or else it is possible to express S in degree of deflection of a galvanometer, if the ray of light, after passing through the image is made, some way, to generate a current in the circuit that causes the deflection of the needle of the galvanometer. In these cases, when the darkening of the plate is of interest as such, as in the studies of the peculiarities of the plates, the darkening is expressed in so-called densities D:

$$D = \lg \frac{I_o}{I}$$

where I_o - the intensity of the radiation that affected the plate, and I - the intensity of the radiation that passed through the darkened place of the plate (See Chapter III).

The degree of darkening S depends not only on I and on t the length of the exposure, but also on the wavelength of the ray impinging on the plate. The range of sensitivity of the place, i. e. the range of wavelength that excites a noticeable S , exceeds the range of the sensitivity of the eye. On the end of short wavelengths almost all plates go beyond the spectrum that has passed through the terrestrial atmosphere. On the end of long wavelengths, with the aid of specially sensitized plates that are on the market, it is possible to reach 0.850μ. In order to form an opinion of a relative spectral sensitivity of a given plate we can study a certain function V(λ), defining it in the same way as we have done for the eye.

$$V(\lambda) = \frac{I\ \lambda max}{I(\lambda)} \qquad (6)$$

where I(λ) is the intensities of radiation of different wavelengths, that cause equal darkenings of the plate, I(λ max), the value of I(λ) with corresponding to the maximum sensitiveness of the plate.

Table 3 contains the values of the function V(λ) for the three basic types of plates: 1) common plates used in astronomical practice. 2) the so-called ortho-chromatic plates, sensitized for yellow light and 3) the panchromatic plates, sensitive to all wavelengths of the visible spectrum. From this table it is evident, that for all three types of the plates, the of maximum sensitivity is approximately the same and is near 440μμ (near H γ). The range of sensitivity of the regular plates lies within the limits of 300μμ to 520μμ. For these plates a rapid decrease of sensitivity is found at the red end of the spectrum. The orthochromatic and panchromatic plates exhibit a characteristic decrease of sensitivity to green light.

Let us study now the function S (I, t) for a given wavelength. The curve S(I) or D(I) for a constant length of exposure, t=const, plotted against log I, as we know from Chapter III, is called the Characteristic Curve. The lower part of this curve corresponds to an underex-posure, and the upper part to an overexposure. For densities that are distributed on the middle, almost straight part, of the Characteristic Curve, the most accurate photometric measurements are obtained, as here to the measured increase of density corresponds the minimum increase of I. The larger the angular coefficient of this straight part of the curve is (usually marked by γ) the more contrasting is the given plate. By assuming different values of t, a related group of characteristic curves can be obtained. The measurements show, that for the majority of the plates, the curves of this group, within a comparatively wider interval of t (t may change by a factor of several tens) remain parallel to each other or, in other words, for a given S, the inclination of the characteristic curves is the same. This means that the function S should appear as S(I, f(t). Schwarzschild, on the basis of many experimental studies, has found that f(t) $=t^P$ (P - is a constant, known as the Schwartzschild Index). The law of Schwartzschild means that for equal densities It^P=const. For the majority of plates P=0.8, but it may vary by 0.1 or 0.2 even for the same plate. The

Schwartzschild formula also means that the curves S(log t) with I=const should have the same form as the characteristic curves with t=const (see Chapter III, §17).

A study of a group of characteristic curves S λ (I) with t=const, for different wavelengths, reveals that these curves are parallel only for neighboring wavelengths and then only in then straight part. The deviation from parallelism indicates, that the ratio of the intensities of different wavelengths, needed to produce equal darkening, changes with the degree of this d darkening. This phenomenon is known as the photographic Purkinje effect owing to analogy with the corresponding effect of visual spectrophotometry. The existence of the Purkinje effect means that the function V(λ), introduced above depends on the density of the image, and therefore the characteristic of the sensitivity of the plates, given in the table are only approximate and refer only to certain mean densities. The amount of the Purkinje effect depends on the brand of the plates. For some plates, the characteristic curves remain parallel for, almost, the entire length of the spectrum. For other plates the difference of the stellar magnitudes m - m , with S=const may fluctuate between the limits of 0.5m, but for practically all plates, the characteristic curves remain practically parallel, within the interval of 10 - 20μμ. This interval of wavelength, may be widened in the part where the V(λ) is more stable, and can be reduced for the wavelengths corresponding to a rapid decrease of sensitiveness.

The curves S λ(t) with I=const demonstrate an effect quite analogous to the Purkinje effect. A small deviation of this effect from the Purkinje effect indicates that the Schwartzschild Index changes somewhat with the wavelenght. In certain cases the quantity P changes by 0.2 with the variations of the wavelengths. So, according to Tikhoff, for some plates, the Schwartzschild Index for the violet equals 0.7 and for the yellow-green 0.9.

The function S λ(I, t), in addition to the three basic parameters I, t and λ, depends also on a number of other factors. For example, the form of the characteristic curve changes with the variations of the temperature of the plate during the exposure. At low temperatures the sensitivity of the plates and the gradient, λ , decrease. According to Barabasheff, the deviations of the characteristic curves from the normal form at room temperature may be so great that a required condition in all photometric studies is that the exposure of all plates intended for a comparative study shall be made within the temperature interval of 10° - 15°.

The form of the characteristic curves depends also, to a large degree, on the development. An important part is played by the type of the developing agent it, concentration, the duration of the process of development and by the temperature of the solution. Therefore, in all accurate photometric studies it becomes necessary to develop all the plates, intended for a comparison, at the same time, and under identical conditions. Also, due to the existance of the effect of staleness it is desirable that for all negatives, the time elapsed from the moment of the exposure to the time of development, is approximately the same.

Table 3

in μμ Plate	340	350	360	370	380	390	400	410	420	440	460	480	500	520	540	560	580	600	620	64
. Supersensitive . Brille)	0.16	0.22	0.28	0.44	0.66	0.81	090	0.95	0.98	1.00	0.97	0.86	0.41	—	—	—	—	—	—	—
Rapid Chromatic . Tikhonsff)	—	—	—	—	—	—	—	0.60	0.73	0.98	0.89	0.36	0.09	0.06	0.12	0.11	0.03	—	—	—
Soft Grad. (Acc. Tichonoff	—	—	—	—	—	—	—	0.63	0.85	1.00	0.85	0.38	0.16	0.13	0.14	0.15	0.16	0.16	0.14	0.1

Usually, in spectrophotometric studies the spectrum of the star to be measured is produced by an intermittent exposure ("widening" of the spectrum) with the aid of a scale, produced by a continuous exposure. In this connection, the problem of the influence of an intermittent wxposure on the density of the image, and on the form of the Characteristic Curve, becomes of interest. The investigations of Fessenkoff have demonstrated, that with a single interruption in the exposure, the resultant densities are the same, within the errors of measurements, as in the case of a continuous exposure. The intermittent exposure may produce a result of a different nature, but probably the for of the Characteristic Curve will remain practically the same.

The peculiar characteristics of the plates vary not only with different bands, but even with individual plates taken from the same package. This makes it imperative to produce all the negatives, intended for a compriosn, on the same plate, or on the pieces cut out of the same plate. But the sensitivity may vary even within the limits of the same plate, and cause an error of measurement of the order of several hundredths part of a stellar magnitude. These plate errors are often of a systematic nature, and thus, in order to insure the accuracy of the measurement within the hundredth part of the stellar magnitude, it becomes necessary to make several exposures of the same object.

In producing the contours of the spectral lines, the errors may be due to the grain of the plates, to the diffusion of the photographic reaction, to the Eberhardt Effect, and to other photographic effects. In the diapositive plates of low sensitivity the grain is so fine, that it obsolutely is not noticeable in the measurement of the spectral lines at intervals of 0.01 mm. With the increase of the sensitivity, the grain becomes courser, as a rule, and in the case of very sensitive plates, as the Ilford Monarch Plates, it may cause occasional noticeable deviations in measurements.

The Eberhardt Effect, i.e. the effect of a increased density of the edge of an image of a uniformly illuminated surface as compared with the central part, is not of such potential danger in the photometry of the lines, as, according to Valenkoff, this effect is noticeable only in images with sharply defined edges. In the fine grained plates the Eberhardt Effect practically does not exist. The Eberhardt Effect, as well as a number of other photographic effects (as the effect of photographic reflection of the image) depend, to a large degree, on the developing agent. In photometric practice, at room temperature can be recommended.

The fogging of the plate is an important source of errors in the photographic photometry. Photographic fog may be of three varieties: 1) the development fog, 2) the fog due to the action of the light, before or after the exposure and 3) the fog produced during the exposure, due to the presence of scattered light. The first two types of fog can be avoided by sufficient care in handling the plates and by close attention during the development. The third type of fog is actually always present. If the fog is of the same intensity on all the plates being compared we do not need, evidently, any corrections for its effect. This sort of a correction becomes necessary, whenever the fog is not uniformly distributed, or when it is of a different intensity on the plates under comparison. The value of the correction can be determined by a measurement of the darkening of a free part of the plate, as close to the measured object as is practical.

Whenever the Characteristic Curve used for reduction of the measurements, is obtained on a plate free from fog or on a uniformly fogged plate, it becomes possible to compute the fog correction for the exposed plate, by finding on the Characteristic Curve a certain value of the intensity ΔI that corresponds to the density of the fog. If the brightness of the object, derived from the Characteristic Curve is I', then the actual brightness I, evidently will be $I=I'-\Delta I$. This method can be employed only in those cases, when the back-ground of the scale is weaker then the background of the plate with the image under measurement, but this case is the one most often encountered in the astronomical practice.

If the fog on the plate with the scale, used for the construction of the Characteristic Curve, is not uniform, it becomes necessary, first to reduce the constructed curve to a uniform background. This reduction can be accomplished by the method of successive approximations. By the constructed curve it is possible to determine a certain correction of the scale ΔI, corresponding to the deviation of the background from a certain mean (background). After this correction is introduced into the scale, a new Characteristic Curve is constructed; by this curve a correction ΔI is determined once more, ett.

The law of summation of the intensities, that underlies this method of reduction, does not excite any doubts, whenever the light that produces the background and the light of the object are of the same color. There are certain indications, based on experimental data, that the fog from red rays may in some cases, even decrease the density of the image, although this

effect is quite negligible.

The law of the summation of intensities can be used also for the measurement of the fog of the second type, because, as we have seen above, the interruption of the exposure does not produce any important effect on the density, that is finally obtained. It seems, that the measurement of the fog of the first type, also should be performed by this rule. Really, the action of the light on the plate, causes a certain decomposition of the grains, that proceeds, in the first approximation, according to the law of a simple chemical reaction. It is very probable, that the decomposition of the grains, that causes the appearance of a chemical fog, proceeds according to the same law.

In those cases, when it is not convenient, to measure the fog by reduction of the intensity of the darkening of this fog, it is possible to compute the correction for the background by the rule ~~of~~ that the summation of the density of the image is proportional to the number of the decomposed grains. If the decomposition of the grains follows the laws of a simple chemical reaction:

$$dn = K\,I\,(n_0 - n)dt,$$

then we have this formula for the density, as to the function of the intensity and the exposure time:

$$D = D_0\,(1-e^{-kIt})$$

Generally, this formula represents the characteristic curves. Also, it expresses the Schwartzschild Law with $P=1$. If kIt is small, this formula means that $D \sim I$. Thus for low densities, the summation of the intensities may be replaced by the summation of densities. Therefore, for computing the density of the object's image without the background, it is necessary to deduct from the measured density the density of the fog. This operation, however, becomes absolutely unpermissible for high density of the image or of the fog.

Although the methods just described permit the introduction of a correction for the fog, this reduction usually leaves a feeling of uncertainty concerning the results. In accurate photometric measurement it is better to avoid the necessity of introducting corrections for the background, by means of the elimination of this background.

From all that was said in the paragraphs above it follows, that spectrophotometric comparisons with the aid of a photographic pl te, can be made only by comparing the darkenings produced by the intensities of the radiations of some wavelength. These comparisons can be made with the aid of a photometric scale, prepared in a form of graded darkenings. The graduation of the darkenings may be achieved by, either changing the intensity of the source of radiation by a certain factor, or, less accurately, by changing the time of the exposure. In the latter case, during the subsequent work it will become necessary to employ the Schwartzschild Law, Itp = const, or some other empirical formula.

The scale and the objects under comparison, obtained in the same wavelength, must be exposed on the same plate, under the same conditions, and as far as possible, at the same time. The development of the plates also must be performed at the same time and under similar conditions. Any deviations

from these rules are permissible only after specially performed investigations, that have confirmed the legitimacy of these deviations.

§5 - The Standardization of the Plates -(Construction of the Spectrophotometric Scales).

The methods of photometric standardization of the plates, can be separated into two principal groups: 1) The obtaining on the plates the darkenings corresponding to a definite gradation of the intensities, and 2) the obtaining of different darkenings from a source of a uniform brightness, by way of varying the exposure time.

The methods of the first group permit the direct determination, of the r required correlation $S\lambda(I)$. It is possible to create a gradation of intensities in every wavelength, by obtaining a number of spectra by, either decreasing the brightness of the studied object, or of the auxiliary source of light by a known factor. For this purpose, in work with slit spectrographs, the use of the "step filter" is most convenient. The "step filter" is a plate containing strips of different tranparency, of about 0.5 mm in width, and spaced regularly. The spaces between the strips are usually blackened. It is desirable, that all these strips are neutral, i.e. that the absorption of light in them is equal for all wavelengths. Such a filter can be made, for example, on a photographic plate, by photographing the source of light with different exposures.

The difference of the stellar magnitudes of the intensities passing through two adjoining strips of the filter usually is selected of the order of 0.3 - 0.5 stellar magnitude.

The step filter is applied directly to the slit of the spectrograph in such a way that the strips are perpendicular to the slit.

During spectrophotometric studies, in addition to the spectra of the observed stars, an additional exposure is made of an artificial source of light, that is made to illuminate, uniformly, the slit of the spectrograph with the filter. This additional plate presents a series of narrow spectra and by messuring their darkening it is possible to construct (if the graduation of the transparency of the strips is known), the Characteristic Curves for every wavelength.

The determination of the gradation of the transparency of the strips of the filters, necessitates a special study of the filter. The most convenient, from the practical point of view, and the most accurate method is the following: using a source of light producing a continuous spectrum, a number of spectrograms is made through the filter, with different widths of the slit. Then the intensity of the radiation, that passes through the filter and the slit will vary, with every exposure, propartionally to the width of the slit. During the successive exposures it is important to control the constancy of the source of light - an electric globe, by watching, for example, for the constancy of the current in the circuit. The series of photographs, obtained in this way, allows the determination for every wavelength, of the intensities of

different strips of spectrum relative to a certain strip of the filter. In the work with the filter it is also necessary to compute a correction for the possible wedge like form of the slit. This study of the filter is done preferably with the same spectrograph that is habitually used with the filter. If the filter is prepared on a photographic plate, this study has to be repeated in the course of 1 - 2 years due to a possible fading of the image.

The standardization of the plate for the spectrophotometric studies at the Pulkovo Observatory is performed by the filters in the way just described.

The wedge method of standardization, developed by Plaskett, and applied by him to the spectrophotometric studies of the Sun and stars, with a slit spectrograph, is a certain variation of the method of the step filter. The wedge is made from a natural glass of low transparency, with its thickest part about .2 m m. By joining it with a transparent glass wedge of the same angle, the wedge is given the form of a flat plate with parallel sides.

During the spectrophotometric investigations, the wedge is placed along the slit of the spectrograph. The wedge may be used in the same way as the step filter, after its preliminary study in the laboratory, i.e. after the determination of the coefficient of the transparency of the wedge p and the angle of inclination (the quantity $p_\lambda \, tg \alpha = \sigma$ is called the constant of the wedge).

On the photographic plate, the distance h, from the bright edge of the spectrum to that part of it, that was produced by the passage of the light through a certain point of the wedge will correspond to the distance mh on the wedge (m is the ratio of the focal length of the collimator to the focal length of the camera). Strictly speaking, due to the chromatic aberration of the camera's objective, and to the chromatic image of the star, m will be a certain function of λ. The intensity of the spectrum at a certain distance h from the edge is found by the formula

$$I_\lambda = k_\lambda \times 10^{\sigma_\lambda mh} \qquad (7)$$

where k_λ is the intensity of the edge of the spectrum. However the use of the wedge in the same manner as the step filter, does not offer any advantages over the use of the filter. By substituting the wedge for the filter it is possible to introduce a number of errors, due to the inaccuracies in the determinations of h and from a possible dispersion of light in the material of the plate, that will blur the true distribution of the intensity along the width of the spectrum.

The advantage of the wedge lies in the quicker and more convenient interpretation of the results in those cases, when the distribution of the energy in the continuous spectrum of the star is being studied. In these studies the image of the star is stretched by a cylindrical lens, thus a uniform lighting of the wedge is attained. Then, after obtaining through the wedge the spectrum of the studied star, as well as that of the reference star, the measurements are made, for each wavelength, at an arbitrary distance h on one plate, the distance h' on the other plate, of that part of the spectrum, that has the same density as on the first plate. Then, as per formula (7) the relative intensity of the two parts of the spectra of the two stars is found from the relationship:

$$\frac{I_\lambda}{I'_\lambda} = 10^{m \sigma_\lambda (h-h')}$$

If the measurements are taken, along the whole spectrum, for the same density, it is evident, that h will be a function of the wavelength. The measurements h and h' can be taken, for example, on a spectrocomparator, after changing in it, for the sake of a convenience, the position of the silvered parts of the Brodhun Prism.

In the spectro-photometric studies with an objective prism, the most widely used method of Standardization of the plates, is the Hertsprung Method.

A wire grating is placed in front of the objective prism in such a way, that the dispersion of the grating and the dispersion of the prism act perpendicularly to each other. Evidently this demands that the wires of the grating are placed perpendicular to the refracting edge of the prism.

The photograph, in addition to the regular spectrum that is produced in exposures with an objective prism, will have auxiliary spectra of the same dispersion as the central spectrum. Each of these spectra corresponds to a different order of the grating spectra. The influence of the dispersion by the grating will manifest itself, evidently, only in a slight distortion of the spectrum: the red ends of the spectra are moved further away from the central spectrum, than the violet end.

If we assume the width of the wire, forming the grating to be equal to d, and the width of the opening between two adjoining wires as S, then the distance between the n-th auxiliary spectrum and the central spectrum at wavelength   , is computed by the formula

$$D=f\frac{n\lambda}{S+d} \qquad (8)$$

where f is the focal distance of the objective or of the reflector's mirror.

On the basis of the constants of the grating, i.e., the quantities S and d, it is possible also to compute the relative intensity of the auxiliary spectrum of any order to the intensity of the central spectrum. This ratio is the same for all wavelengths and may be computed by the formula mentioned in Chapter III.

The regular gratings are chosen in such a way, that the gradation of the intensities is as close, as possible, while the distance D is large enough to avoid a possible overlapping of the spectra during the widening at the time of the exposure.

In the commonly used gratings S and d are equal. Hertsprung, for instance, used gratings with S=d=0.5 m m. For such gratings the spectra of the even or orders disappear, and only the spectra of odd orders remain. The intensity of the first auxiliary spectrum is less than that of the central spectrum by, approximately, one magnitude.

Farther on, the intensity changes inversely as the square of the order.

The main disadvantages of the grating method are: 1) An excessively rapid fall of the intensity from spectrum to spectrum that results, under the ordinary conditions, in only two auxiliary spectra of some intensity on both sides of the central spectrum. For the construction of the characteristic curves, this furnishes only 2 or 3 points; and 2) the distortion of the spectra that adds to the difficulty of their interpretation.

Inasmuch as the photometric scales ~~save~~ serve only for the determination of the peculiarities of the plates, it is evident, that the gradation of the darkening, may be obtained with the aid of another instrument, as long as it is made on the same plate, or on a piece of the same plate. Thus, for example in observations with the objective prism we may produce the scale on a part of the same plate, by means of a step filter mounted on a laboratory slit spectrograph. This method of standardization is much more convenient, than the Hertsprung Grating Method.

It is also possible, although not so accurately, to make a scale by photographing with a tube photometer. In order to diminish the inaccuracies that arise, due to the effect of Purkinje, it is necessary to photograph the tube photometer through a light filter, that passes the rays of a wavelength, near to the studied wavelength. This was the technique employed by Ching Sung Yü during his studies of the Hydrogen absorption in the stars of the types B and A.

The methods described above permit of producing the whole photometric scale simultaneously. In some cases are used the methods of standardization, based on obtaining a number of consecutive photographs of the spectra of the stars. These consecutive photographs are obtained with the intensity of the spectra being gradually reduced in known proportions. This can be effected by reducing the apperture with an objective prism, by applying the diaphragm to the prism. The ratio of intensities are computed in these cases as the ratio of the unobstructed areas. For elimination of the errors, connected with the variations of transparency of the objective along the radius, diaphragms of a sector type are usually employed, the opening, of these diaphragms being sectors of different angles. The errors, due to the varying transparency of the objective prism, are avoided by the use of a diaphragm, consisting of a row of strips, placed perpendicularly to the refracting edge of the prism.

The most serious disadvantage of these methods is the fact, that the photographs are not taken simultaneously. In the interim between the exposures, the transparency of the atmosphere may change, and this is a factor that is very difficult to take into account.

Under certain conditions it is possible to construct the characteristic curves from a single photograph of the spectrum of the star, without any auxiliary scale. This is possible, when, for example, the distribution of energy within certain spectral lines is known, or if the relative intensity of some lines is known. Such a method of study of the intensities of spectral lines on the spectrograms, obtained without a scale, was proposed by Hogg.

Usually, while photographing the spectrum of the star the spectrum of an artificial source as of an iron arc is also recorded. Such a plate can be calibrated, as the relative intensity of the lines of the spectrum of iron is known, but it is important to choose those spectral lines that exhibit only small variations with a change of the physical conditions.

In the studies of the contours of the spectral lines with the aid of an objective prism, it is possible to compute the characteristic curves by a comparative study of the spectra of the stars of known and different integrated intensity. If the selected stars belong to the same spectral type, it may be assumed, that the relative intensities of different parts of the spectra, are approximately proportional to the integrated intensities of the star.

The methods based on the variation of the length of the exposure are comparatively rarely used in spectro-photometric studies. Rosenberg, during his study of the distribution of the energy in the continuous spectrom of the star, with an objective prism, has obtained a scale, by making several photographs of the spectrum of the reference star with different exposures, whose length varied in a geometric progression. The reference star was photographed, as near the meridian, as possible, in order to minimize the changes of the coefficient of the transparency of the atmosphere. In order to avoid the errors, that can arise due to the uneven widening of the spectrum, the stellar spectra were widened not by the means of moving the image of the star, but by placing the plateholder out of focus.

Unsöld, during the study of the intensities of the spectral lines of the Sun with a slit spectrograph, used a rotating sector, that was, really, a disc with openings placed along a sector. The width of these openings changed in steps in the direction of the radius of the disc.

The sector is placed directly over the slit, and during the exposure the disc is rapidly rotated. On the obtained photograph, the intensity of the spectrum changes, by steps, along its height. The gradation of the intensities corresponds to the gradation of the exposures, that are proportional to the widths of the openings of the sector. The use of this method meets with serious objection, owing to the intermittency of the exposure.

The employment of this method, as well as of Rosenberg's should be accompanied by the computation of the Schwarztschild's Index, for the change from K to I.

§6 The Photometric Measurements of the Spectrograms and the Methods of the Construction of the Characteristic Curves.

The basic apparatus, used for making the measurements on the plates, is the microphotomer of Hartmann, described in detail in the Chapter III. In order to measure a spectrogram by the Hartmann's microphotometer, the plate is placed on the frame in such a way, that the direction of the micrometric movement coincides with the direction of the spectrum. The divisions on the drum, that turns with the movements of the plate are marked in 0.01mm.

With the aid of a very narrow, rectangular mirror, placed in one of the cubes of the Lummer-Brodhune Prism, a slit like portion of spectrum, about 0.02 m. m. wide, is cut out. The visible width of this part of spectrum is sufficient to allow a convenient comparison of its brilliancy with that of the background. A cure should be exercised, that this slit-like portion of the spectrum is strictly perpendicular to the direction of the spectrum. The equalization of the (intensities of the) part of spectrum and the background is done by the wedge, and a reading of the position of the wedge during the equalization is taken.

The measuring the distribution of the darkening within the spectral lines of normal width several consecutive readings are taken, with the plate being moved every 0.01 mm. Also, the comparisons with the scale are made, as well as the comparisons with the background, in order to introduce a correction for fog. The readings taken during the comparisons with the scale, supply the data for the construction of the characteristic curves for every wave-length. With such a curve it becomes possible to make the measurement of the darkening of certain points of that part of spectrum, whose intensity is being studied. The accuracy of the readings on a Hartmann Microphotometer, is very high- it is of the order of 0.01 - 0.02 stellar magnitude for finegrain plates.

The occasional, small defects of the plates are not very important during the work with the Hartmann Michrophotometer, as during the comparison of a narrow part of the spectrum with the background, it is possible to detract the eye from the effect produced by the defect of the plate.

The obtaining of the contour of lines in this way is very tiresome, as between 100 and 200 readings are made for the wider lines. Much quicker results, although not always of the same accuracy, are obtained with the so-called registering microphotometers, but for the photometry of the continuous spectrum, that necessitates obtaining measurements of comparatively, few points, the Hartmann's Microphotometer is the most convenient apparatus.

In the registering microphotometers, the eye is entirely eliminated from the measuring process. These microphotometers may be divided into two groups: those with a thermo-couple, based on the thermo-electric effect, and those utilizing the photo-electric cell, based on the photo-electric effect. The representative apparatus of the first group is the Mull microphotometer, while the Koch microphotometer represents the second type.

A schematic drawing of the Mull microphotometer is shown on the fig. 59. The light from an incandescent globe L, after passing through the condensator C falls on the slit S_2. With the aid of the objective O_2, an image of this slit is projected on the studied photographic plate N. A magnified image of that part of the plate, covered by the image of the slit S_2, is in its turn projected on the second slit S_1 by the objective O_1. The diaphragm, bearing the slit S_1 covers the thermo-element V. The width of the slits S_1 and S_2 may be controlled in the same way as is done in spectroscopes. The length of the slit S_2 can be also changed.

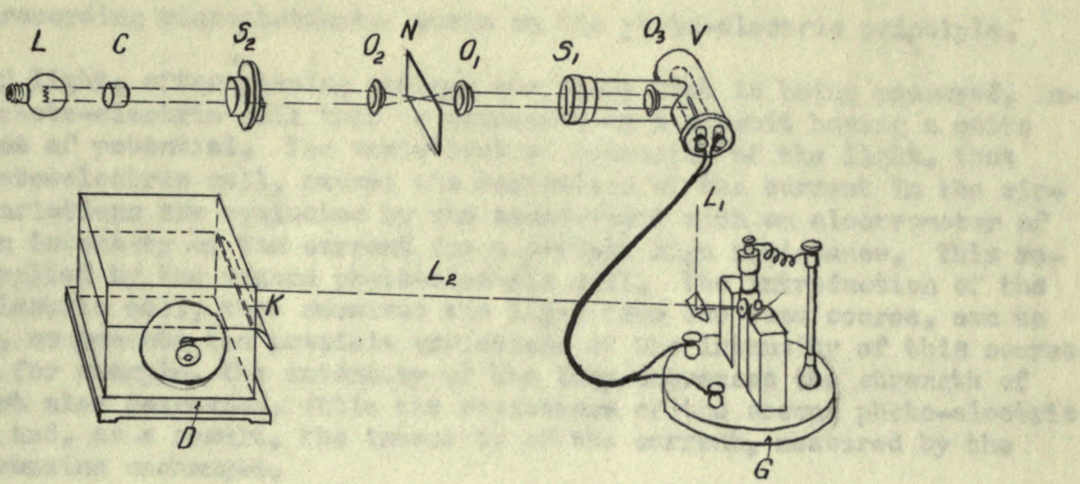

Fig. 59.

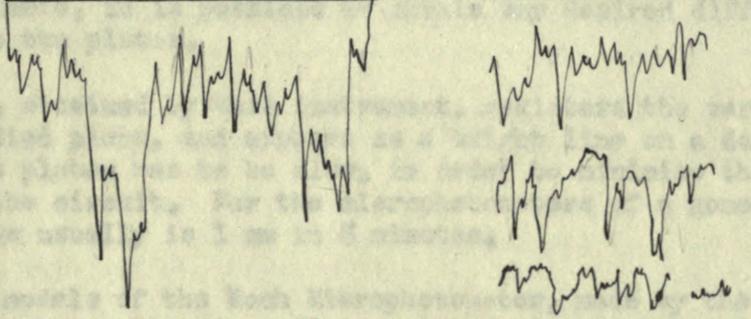

Fig. 61.

Such photograms are measured by placing over them a glass plate with an engraved rectangular grid, whose squares are of 1mm. After measuring the photogram of the scale, it becomes possible to construct the characteristic curves, that relates the measurement of the grid to the intensities. From this curve it is possible to compute the contours of all measured lines, expressed in intensities.

The Koch recording microphotometer works on the photo-electric principle.

The ray of light, after passing through the plate that is being measured, impinges on the photo-electric cell that is connected in a circuit having a quite large differance of potential. The variations of intensity of the light, that strikes the photo-electric cell, causes the variations of the current in the circuit. These variations are evaluated by the measurement with an electrometer of the decrease in intensity of the current for a certain high resistance. This resistance is supplied by the second photo-electric cell. The introduction of the second photo-electric cell, that receives the light from the same source, can to a large degree, compensate the possible variations of the intensity of this source. Actually, when for example, the intensity of the lamp decreases the strength of the photo-curent also decreases, while the resistance of the second photo-electric cell increases and, as a result, the intensity of the current, measured by the electrometer, remains unchanged.

During the measurements, the plate is placed on a table, unlike the table of the Hartmann Photometer. The rays from a bright light, after passing through the plate, pass through an optical system, analogous to that of the Hartmann Microphotometer, and reflecting from the mirror of a Lummer-Brodhun Prism, impinge on the photoelectric cell, thus causing the deflection of the thread of the electrometer. By means of an optical assembly, the shadow of this string is projected, as a dark line on the cylindrical lense, fitted into a suitable opening in the lid of the box, containing a photographic plate, on which the photogram is recorded.

During the measurement, an electric motor moves, slowly, the measured plate and the plate on which the deflection of the electrometer are recorded. As with the Mull Microphotomete, it is possible to obtain any desired difference between the scales of these two plates.

The photogram, obtained by this instrument, registers the variations of the density of the studied plate, and appears as a bright line on a dark background. The movement of the plates has to be slow, in order to minimize the possible effect of the inertia of the circuit. For the microphotometers of a general type, the speed of this motion usually is 1 cm in 5 minutes.

In the newest models of the Koch Microphotometer, made by the Zeiss Company of Jena, the second photo-electric cell is omitted for the sake of decreasing the inertia of the circuit, and replaced by a common resistance coil. In the Zeiss Microphotometer, the photograms are obtained just as in Mull apparatus, with a dark line on a bright background. This allows several recordings to be made on one plate.

The next step in the study of the plate, is the expression of the darkenings,

recorded by one or another microphotometer, in terms of intensities. The intensities are computed, as was mentioned before, by the characteristic curve plotted according to the darkening of the scale. In as much as these scales have few graduations, the characteristic curves are usually plotted by a few points - usually 5 to 10. During the spectrophotometric measurements, especially when the spectra of wide dispersion and containing many spectral lines, are studied, or else, when the plates have many scales obtained from several stars (as by Hertzsprung Method with an objective prism), always it becomes possible to increase the number of points plotted on the characteristic curve. In fact, in those cases it is possible to plot several characteristic curves, corresponding to different densities of the same gradation. If these curves are plotted for the same plate and the same wavelength, they should be parallel. By transferring to one of these curves the others by the method of a parallel shift, to the point of the best coincidence, it becomes possible to plot the characteristic curve with many points, and quite often in an increased interval of log I.

The graphical method of plotting the characteristic curves may be replaced by a computation. This method was followed by two Greenwich Observatory Astronomers Greaves and Davidson during their studies of the distribution of energy in the continuous spectra of stars, with an objective prism.

We may attempt to express the characteristic curve by the following equation

$$\lg I = \alpha + bS + bS^2.$$

By substituting in this expression the measured values of the darkenings of the steps of the scale and the known relative intensities, it becomes possible to determine by the method of least squares, the coefficients b and c. The coefficient a disappears during this operation, but it does not present any interest, as the ratio of the intensities is always computed, i.e. the difference of the logarithms.

In those cases, when the scale consists of two steps only and when it is not desired to use the analytical method of determination of the characteric curve, it is possible to use the method proposed by Scwarztschild, and allowing the construction of the curve to be performed in a more exact way than is possible by a simple parallel shift of the straight line segments, plotted from two points. We shall study this method in an example of a part of spectrum containing an absorption line.

Let us assume, that at a certain point of a spectral line of one spectrum we have a darkening S_1, while at the corresponding point of the same line on the second spectrum which corresponds to the second step of the scale, we have a darkening S_2. To this corresponds a certain increase of intensity expressed, say, in stellar magnitudes Δm.

Then, on the first spectrum is found a point on the line, with a darkening S_2; to this point on the second spectrum will correspond a darkening S_3 which should be, evidently, of intensity $2\Delta m$.

By continuing this process it becomes possible to determine many points corresponding to the function S (lg I).

Although the described methods allow the characteristic curves to be constructed from many points, while only a few steps of the scale are known, one must endeavor to determine a large number of the steps of the scale, as those methods, as all chain methods, generally, bring about an accumulation of errors.

§ 7 - The Photometry of the Continuous Spectrum. The Measurement of the Temperatures of the Stars.

The problem of the photometry of the continuous spectrum is the measurement of the intensities of those parts of the spectrum, that do not contain any spectral lines, and thus computing the curves of the distribution of energy in the stellar spectra. These intensities can be determined by a photometric comparison of the studied spectrum with a spectrum of a certain standard source, at the same wavelength.

If the function of the distribution of energy $I_0(\lambda)$ in the spectrum of the reference star is known, it becomes possible to find the function $I_1(\lambda)$, that defines the law of the distribution of the energy in the studied spectrum.

When the form of the function $I_0(\lambda)$ is unknown, it is possible to derive only a relative curve of the distribution of energy $\frac{I_1(\lambda)}{I_0(\lambda)}$. The observations have demonstrated, that for many stars, in the observable part of the spectrum, the curve $\lg\frac{I_1(\frac{1}{\lambda})}{I_0(\frac{1}{\lambda})}$ has a straight line form. Owing to this it becomes convenient, especially for statistical reasons, to replace the tables of the values of function $\frac{I_1(\lambda)}{I_0(\lambda)}$, by a single number, G - the angular coefficient of the mentioned straight line. G is usually called the relative spectrophotometric gradient. So

$$G = \frac{d\lg_e\frac{I_1(\lambda)}{I_0(\lambda)}}{d(\frac{1}{\lambda})} = 0.922\frac{d\Delta m(\lambda)}{d(\frac{1}{\lambda})} \qquad (9)$$

where $\Delta m(\lambda)$ is the difference, in stellar magnitudes, of the intensities of the two parts of spectra, of the same wavelength that are being compared, viz.,

$$\Delta m(\lambda) = m_0(\lambda) - m_1(\lambda).$$

Quite often the quantity G varies with the wavelength and, therefore, it is necessary to mention the interval of the wavelengths, that corresponds to the given gradient. From the definition of the gradient it follows, that two systems of gradients that are based on two different reference stars differ only by a constant which equals the difference between the gradients.

The distribution of energy in the spectra of the stars, as follows from theoretical considerations as well as and as proved by observations, generally speaking, differs very little from the case of equlibrium radiation, expressed by Planck's

formula

$$I(\lambda) = C_1\lambda^{-5}\left[e^{\frac{C_2}{\lambda T}} - 1\right]^{-1} \qquad (10)$$

where C_1 and C_2 are certain constants and T is the temperature of the equlibrium radiation (C_2 =14350μ. grad). In application to the stellar spectra, T may be regarded as a certain parameter, chosen in such a way, that formula (10) expresses in the best way, the distribution of energy in the spectrum of the star. A value of the quantity T, thus obtained, is called the spectrophotometric effective temperature of the star.

The effective temperature presents a very convenient characteristic of the distribution of energy in the spectrum of the star. However, the most important meaning of the effective temperature is that, in the first approximation, it represents the actual temperature of the outer layers of the star.

In exact studies of the distribution of the intensities in the spectrum of the star, by formula (9), it is not possible to present the function I(λ) of the star over a large interval of the spectrum. However, for small intervals of the wavelengths, it is always possible to choose a value for T in such a way, that formula (10) corresponds well to the observations. In such cases T will change somewhat with the wavelength. Therefore, in exact studies it is necessary to mention the interval of wavelength used in the computations of the effective temperature.

For small wavelengths and for low values of T, instead of the formula (10) it is possible to use a simpler one, Wien's formula:

$$I(\lambda) = C_1\lambda^{-5}\, e^{-\frac{C_2}{\lambda T}} \qquad (11)$$

From formula (11) it is evident, that the error in the computation of the effective temperature increases with the temperature T^2. For very high temperatures Planck's formula transforms into the Law of Rayleigh-Jeans:

$$I(\lambda) = C\frac{T}{\lambda^4} \qquad (12)$$

In this case the determination of the temperature of the star from the form of the curve I(λ) becomes impossible, as T, being a constant factor, is eliminated. For stars with temperatures over 20000°, 30000°, as, for example, the stars of the Wolf-Rayet type and the noval, the determination of the temperatures by the spectrophotometric method, becomes practically, impossible.

It is easy to produce a formula, that allows the effective temperature to be computed by the spectrophotometric gradient. By applying Planck's equation both to the studied star and to the reference star, we have

$$G = \text{const} - \frac{C_2}{T}\; 1 - e^{-\frac{C_2}{T}}\;{}^{-1}. \qquad (13)$$

In this formula the constant depends on the temperature T_0 of the reference star

$$\text{const} = \frac{C_2}{T_0}\left[1 - e^{-\frac{C_2}{\lambda T_0}}\right]^{-1},$$

The strictly photometric device is the system of Nicol prisms N_2 N_3. The Nicol N_2 may rotate around the axis while the Nicol N_3 remains stationary; the angle of rotation is read upon the circle of intensities to an accuracy of 0°.1. The brightness of the artificial star will change according to Malus' law. The indicator of the circle is so arranged that for Nicols set for complete extinction the reading is zero. Then the change in brightness will take place according to the following formula:

$$I = c \sin^2 \varphi .$$

In Zellner's photometers of the newest constructions there is a side eyepiece for observations of bright stars, and there is also a device for the registrations of the readings on the circle of intensities on the paper ribbon. Previously, a kerosene burner whose flame remains constant for several hours was used as a source of light. At present incandesant lamps are used for this purpose, and the ~~circuit-force~~ is controlled by a milliammeter. Figure 42 shows one of the new photometer models made by Toepfer.

The computation of the observations are made by the formula:

$$\Delta m = -5 \left(\log \sin \varphi_1 - \log \sin \varphi_2 \right) \qquad (13)$$

where φ_1 and φ_2 are readings of the circle for two stars, and Δm is the difference of their magnitudes which is to be determined.

The equalization of the brightnesses of the natural and artificial stars must be made at four positions of the circle of intensities in order that the errors depending on the zero point of the circle and on the eccentric position of the Nicol prisms may be eliminated. Let us assume that the following readings were obtained: $\alpha_1, \alpha_2, \alpha_3, \alpha_4$, Let us denote by α_0 the zero-point, i.e. that reading of the circle at which the artificial star disappears, and let us denote by ε the error of eccentricity. The circle of intensities is usually so divided that the subdivisions go from zero to 90° on both sides and then go from 90° to 0° again. Under these circumstances the correct reading will be expressed as follows:

$$\varphi_1 = \alpha_1 + \varepsilon_1 - \alpha_0 \quad \varphi_3 = \alpha_3 + \varepsilon_3 - \alpha_0$$

$$\varphi_2 = \alpha_2 + \varepsilon_2 + \alpha_0 \quad \varphi_4 = \alpha_4 + \varepsilon_4 - \alpha_0$$

Taking into consideration that

$$\sum \varepsilon = 0,$$

we obtain the following true reading:

$$\varphi = \frac{1}{4} \left(\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 \right).$$

by presenting to the eye a complete curve of the distribution of the darkening in the spectrum, the selection of the parts of spectrum, that are most free of the lines and their wings.

The dispersion of the temperatures within a spectral type, also may cause discrepancies in the systems of temperature if they are based on different groups of the stars. For the same spectral type, the spectrophotometric temperatures may differ, depending on the absolute magnitude of the star (so called magnitude), t.e. depending on the fact whether the star is a dwarf or a giant. Still other stars, as those of the types O and B, have a normal temperatures, that vary greatly form these mean values, which are given in the table. For instance, the star P Cygni, of type B2 has a spectromhotometric temperature of 5700°. In this respect, the stars of the type A0 exhibit the smallest dispersion, and for this reason the stars of this type are taken as the reference stars in the spectrophotometric measurements. At the present, the effective temperature of the stars of the type A0 is assumed to be 10.000°.

In the computations of the relative gradients, evidently, it becomes necessary to introduce a differential correction for the selective absorption of the terrestrial atmosphere. From formula (4a) and from the definition of the gradient (9) it follows, that the correction of the gradient ΔG may be sought as

$$\Delta G = G\ \text{actual} - G_{\text{observed}} = - F(z)-F(z_0)\ \frac{d\ \lg_e P(\lambda)}{d\left(\frac{1}{\lambda}\right)},$$

where z, and z_0 are Zenith distances of the studied star and of the reference star In the majority of cases, this equation may be replaced by the more simple one:

$$\Delta G = A\ (\sec z_1 - \sec z_0)$$

The constant A may be found by observing the same star at different Zenith distances, or from the formula

$$A = -2.303\ \frac{d\lg_{10} P(\lambda)}{d\left(\frac{1}{\lambda}\right)}$$

and from the tables of the coefficient of transparency. From the table of the values of the quantity $P(\lambda)$, given in the 2, it follows, that the constant A for Mount Wilson equals +0.38.

Whenever it is possible, the Sun, whose spectrum is well known, may be used as a reference source. During the study of the distribution of energy in the spectra of the details of the Sun's disc (Sun spots, prominences etc), it is handy to use for a reference, the spectrum of the center of the Sun's disc, or of some other point, whose position in reference to the center is known.

§8 - The Photometry of the Spectral Lines.

The study of the spectral lines reveals the composition of the stellar

atmospheres and the physical conditions, under which the gas, producing the given line, is found.

The form of a spectral line depends on a whole complexity of the characteristics of the state in which the gas is found. Therefore, the study of the spectral lines may furnish a solution of a number of diverse problems of the constitution of the stellar atmospheres.

The fundamental problem of the Photometry of the Spectral Lines is presented by the computation of the values of the function $I(\lambda)$ within the spectral line. In these computations, usually, the intensity of the continuous spectrum in the vicinity of the spectral line is taken for the unit of intensity.

The curve $I(\lambda)$ is known as the Contour of the line. The contour lines, in general, are symmetrical and resemble the Gaussian error curve.

In the stellar spectra, unlike the spectral of terrestrial sources, the spectral lines are sometimes, very wide, and may reach widths of many ten's of Angstrom units. Usually, in the wide absorption lines, the intensity falls very slowly, in the direction away from the center of the lines and forms the so-called "wings" of the spectral lines. If we have the contour of the line, we can compute certain numerical values, characteristic of the given line. From the point of view of a physical interpretation of the lines, the following data is of interest: 1) the total width of the line, expressed in Å, or the, so called, semi-width of the line i.e. its width, to the point, where $I=0.5$ (for the absorption lines); 2) the residual intensity, or the I_{min} and 3) the total intensity of the line $\int(1 - I)\, d\lambda$, expressed in the units of the intensity of the continuous spectrum X Å.

After obtaining a photograph of the studied star along with a photometric scale, the following procedure is followed for construction of the line contour $I(\lambda)$:

By means of a Hartmann Photometer, or of a registering microphotometer, the determination of the distribution of the darkening within the line $S(\lambda)$ or of S, as a function of the linear distance along the spectrum, is made. This curve allows all the darkenings to be expressed in terms of the intensity, the Purkinje effect along the line, not being considered.

If the line is so narrow, that the sensitivity of the plate remains practically the same along its length, then the arithmetic mean of the values of the intensity of the spectrum to the left and to the right of the line, is taken as the unit. For the wide lines, when the variations of the sensitivity of the plate are noticeable, it may be assumed, that the variation of the sensitivity is proportional to the wavelength on the plate.

Owing to the fact that the spectral lines merge into the continuous spectrum very gradually, and to the fact that the stellar spectrum is usually marked by other weak, but numerous lines, the determination of the boundaries of the continuous spectrum is quite difficult. This arbitrariness in the evaluation of the intensity of the continuous spectrum is the most important source of errors in

the computations of $I(\lambda)$.

The plotted lines contours are seldom smooth, but usually consist of separate small rises and falls. The reason for this lies in the granular constitution of the plate but, sometimes it is due to that the basic line, that is measured, is overlaid by several adjoining lines. The effect of the granulation may be lessened by the use of the plates of the finest grain, and by enlarging the dispersion of the apparatus used to obtain the spectrum.

The elimination of the effect of blending of the adjoining lines presents many serious difficulties. This effect is especially strong in the line contour in the spectra of types G - M, as, for example, the solar spectrum.

The smoothing of the contour lines can be done only to a rough approximation by those points within the line, that are most free from the effect of blending, and by utilizing the possible symmetry of the line.

Under laboratory conditions, the following method is employed for resolution of several blending spectral lines. By assuming for every one of these lines the expression of the function $I(\lambda)$ in the form of $e^{-k(\lambda-\lambda_0)^2}$, or else by using a distribution by Hermite Polynomials, and by assuming, further, that the intensities of different lines add up, the coefficients of these functions are found from the observed curve. By this method Hausen has studied the fine structure of H and divided it into three components.

However, this method cannot be applied to the interpretation of the stellar spectra without a preliminary theoretical elucidation, as in the majority of the cases, the intensities of the lines, in the stellar spectra, do not add up.

Besides these principal sources of errors in the construction of the line contours, as well as of the common errors inherent to the photographic plate, a number of other inaccuracies arise, due to the apparatus used to obtain the spectrum.

For the study of the line contour it is possible to use slitless spectrographs (objective prisms), as well as those with a slit. The objective prism has a disadvantage of producing blurred line contours, due to the unsteadiness of the star and the faults of the guiding mechanism (the unsteadiness causes the widening of the image, corresponding to the widening of the opening in the slit apparatus). This blurring of the lines depends on the atmospheric conditions and, therefore, is almost impossible to account for.

The slit spectrographs with prisms are the most convenient for the study of the spectral lines. They produce very little light dispersion, that can distort, quite badly the determinations of the residual intensity of the lines. By working with a constant width of the slit, it becomes possible, by a strictly experimental method, to evaluate the influence of the width of the slit and of the diffraction phenomena on the image of the slit on the plate, through the study of the so-called normal contours of the lines, i.e. of those contours, that would be

produced by a perfect narrow spectral line. Such very thin lines can be produced by terrestrial sources, such as the mercury arc.

Let us assume, that the normal contour is known, and can be represented by the function N(x), where x is the distance from the center of this contour. Let us call I(x) - the observed contour of a certain line, and its actual line - i(x). Then, it is evident, that

$$I(+) = \int_{-\infty}^{+\infty} i(\xi) N(x-\xi)\, d\xi$$

The function i(x) can be found by the solution of this integrul equation of the first type. The solution may be performed best by the distribution of all functions entering into it, by Hermit's Polinomials. Still, in actual practice, this kind of correction should be used only, when the width of the observed line is of the order of the width of a normal contour, t.e. is comparable to the lines of an artificial source of light on the spectrograms.

The spectrographs with the diffraction gratings allow spectra of wide dispersion, to be obtained and they can be used for the detailed study of the structures of the spectral lines. However, owing to the great loss of light, they are used, almost exclusively, in the studies of the Sun. The spectrum produced by the diffraction grating is not ascleancut, as that produced by a prism. From the point of view of photometry, the diffraction grating has a disadvantage of producing a lot of diffused light and also of producing false, or "ghost" lines. The diffused light, produced by a simple reflection from the surface of the grating, is superimposed over the spectrum, and blurs the spectral lines; in some cases its intensity reaches 15 - 20^{0}/o of that of the continuous spectrum. It is very difficult to account for it, but if a line of a known intensity is found near the studied line, then it becomes possible to compute the intensity of the diffused light, through a measurement of the apparent intensity of the known line.

The "ghost" lines are the result of a periodic error of the screw of the dividing machine, that was used to make the diffraction grating. Usually they are placed symmetrically in reference to the spectral line. Their intensity depends on the quality of the grating, and changes with the order of the spectrum, in inverse proportion to the square of the order.

The integrated intensity of the "ghost" lines may reach, under certain conditions, the quite significant order of 15 - 20^{0}/o. The computation of their influence on the contour of the line, may be performed in the following way: by producing a spectrum of a terrestrial source having few lines (as the spectrum of the mercury arc), it becomes possible to measure the intensities of the "ghost" lines α_1, α_2, etc relative to the intensity of the line, and their distances from it - x_1, x_2, etc. Indicating log I(x) the observed contour of the line, and its actual contour by i(x), we, evidently have:

$$I(x)=i(x)+\alpha_1\left[i(x-x_1) - i(x)\right]+\alpha_2\left[i(x-x_2) - i(x)\right]+\ldots$$

or, approximately,

$$I(x)=i(x)+\alpha_1\left[I(x-x_1) - I(x)\right]+\alpha_2\left[I(x - x_2) - I(x)\right]+\ldots$$

this foumula can be used to compute i(x). As is evident from this formula, the effect of the ghost lines is expressed in the washing out of the line, i.e. in the decrease of its depth.

In attempting to summarize all that was said about the errors in the measurement of the intensity of the lines, it is possible to come to the following conclusions, of the three mentioned characteristics of the lines, the most difficult to measure is the residual intensity. The observed residual intensity is, usually, higher than the actual. The width of the line can be measured very roughly, and the observed width is, usually, also larger than the actual. The integrated intensity is measured with the highest accuracy of all, as the various errors of the apparatus, result only in the re-distribution of the energy within the line.

The spectrum, produced by the slit spectrograph, with prisms, allows the most accurate photometric measurements of the lines. Let us note, that the error of reliable measurements of the intensities of the lines reaches 5°/o.

The problem of the accurate measurements of the intensities of the spectral lines has attracted the attention of astro-physicists only during the last decade. However, at the beginning of this century, Rowland had already prepared a cataloque of the lines of the solar spectrum, and evaluated their intensities on a certain arbirtary scale. This Rowland Scale was used later in the measurements of the intensities of the lines of the stellar spectra. In the solution of many problems that do not demand high accuracy, even now there are reasons to use the Rowland Scale. In 1928 the astronomers of the Mount Wilson Observatory calibrated the Rowland Scale according to the number of the effective dispersion centers, corresponding to the given lines. Their data was used in the preparation of Table 5, of the transposition of the values of Rowland into the logarithms of the intergrated intensities, I, for the part of the spectrum near 500μμ.

Table 5

R	lg 1	R	lg 1	R	lg 1
-3	-0.90	4	+1.00	15	+1.77
-2	-0.58	5	1.20	20	1.86
-1	-0.27	6	1.34	25	1.94
0	-0.00	7	1.44	30	1.99
1	+0.26	8	1.50	35	2.04
2	+0.52	9	1.56	40	2.08
3	+0.76	10	1.60		

For other wavelengths, this table changes, somewhat; but it can be used, with an approximation ot one tehth of lg I, for the part of spectrum from 300 too 600μμ.

§ 9 The Measurements of the Infra-Red Part of the Spectrum.

Not long age, specially sensitized photographic plates enabled Babcock at Mount Wilson Observatory, to reach 120μμ, in his studies of the solar spectrum. The use of these plates for studies of the stollar spectra is hindered by their low sensitivity. Due to this fact, the spectrophotometric studies of the infra-red spectrum were not performed with the aid of photographic plates, but with the use of special apparatus, called a bolometer and radiometer, both based on the principle of the transformation of the energy of radiation of the object into the easily measurable thermal energy.

It is easy to see, that these apparatuses may be constructed in such a way, that their sensitivity will remain constant for all wavelengths. Owing to this fact, theoretically, the bolometer and the radiometer allow the penetration, as deeply as desired, into the infra red part of the spctrum, although their sensitivity to the visual rays falls appreciably below that of the eye or of the photographic plate.

In practice, their range is limited only by the transparency of the employed optical system and of the atmosphere.

The principal part of the bolometer is a very thin, blackened platinum strip, that forms, one of the branches of a wheatstone Bridge, through which a weak electric current is passed. The observed part of the spectrum is projected on this platenum strip, and by heating it, changes the resistance of the circuit. The change of the intensity of the current, brings about a declination of the indicator of a galvanometer, connected in the circuit, this declination being the measure of the value of the decrease of the energy.

Langley used the bolometer in his studies of the solar spectrum, as early as 1880. In recent years, the accurate bolometric measurements of the solar spectrum in the visual and the infra red parts were performed by Abbot. The details of these studies, as well as a detailed description of the bolometer used will be found in the fifth chapter of this book.

In 1923, Abbot, Fowler and Aldrich made an attempt to apply the bolometer to the study of the stollar spectra. In spite of the fact that these measurement were made with the 100 inch reflector, the brilliancy of the different parts of the spectra was found to be insufficient for its exact measurements with the bolometer.

The radiometer presents a certain modification of the well known Crooks' "light mills". The most important part of the radiometer is a double blade supported by a thin quartz thread or a silk thread. One of the blades is blackened, while the other acts as a counterweight. The blades one of the radiometer used in the measurements of the stellar spectra, were about 0.5 $m.m^2$ in area, and were made

from the wings of a housefly. The thread with the blades is suspended within a hermetically closed vessel, and the air was evaluated from it to a certain optimum height of a mercury column usually to about 0.02 mm.

The ray of light is directed through a window in the wall of the vessel into the blackened blade of the radiometer. The heating of this blade causes a difference of pressure to develop in the surrounding air, which forces the blade to be deflected in the direction of the rays, through a certain angle, depending upon the intensity of the radiation and upon the tongue of the filament. The angle of deviation is registered with the aid of a minute mirroe, suspended from the filament of the radiometer, which reflects the light from an auxiliary lamp. As predicted by the theory of the radiometer, and as demonstrated by the experiments, the angle of the deflection of the blade of the radiometer is proportional to the energy that was received by the blade.

The sensitivity of the radiometer has been increased from year to year by the introduction of many improvements for example, Nickolson, who made the first measurements of the thermal effect of the stars at the Yerkes Observatory, placed a glass plate near the blades of the radiometer, and thus increased its sensitivity.

The first use of the radiometer in the study of the stellar spectra, was made by Abbot in 1924, on the 100 inch reflector of the Mount Wilson Observatory. In 1928 he repeated his measurements with an improved radiometer, in which the air was replaced by Hydrogen.

The ray of light from the star, passed through a flint glass prism, and was separated into a spectrum of such a dispersion, that its section from 480μμ to 200μμ was about 30 m.m long. From this part of the spectrum, a portion of 0.4 m.m. wide was cut out by a diaphragm, and projected on the blade of the radiometer. The deviations of the blade of the radiometer were registered by a ray of light reflected from a mirror onto a scale, placed in 6 m from the radiometer. The difference between the reading, before and after illumination by the star light, give the criterion of the intensity of the radiation.

The correction for the atmospheric transparency, and for the absorption of light in the optical system of the apparatus, was computed from the observations of the Sun, the distribution of whose energy in the spectrum is well known from the bolometric measurements. In order to make this kind of comparison possible, a system of diaphragms was introduced, decreasing the sunlight 10^{10} times. By this method it becomes possible to compute the distribution of energy in the stellar spectra, expressed in dynamic units.

The spectra of the planets were investigated and those of 17 stars down to the 3d magnitude, in the interval from 480 to 2200μμ. The error of these measurements was proved to be in the order of 10°/o.

Chapter 2. - Indirect Spectrophotometry (Colorimetry)

§ 10 - The Basic Principles of the Colorimetry.

The exact spectrophotometric measurements of the stellar spectra, as may be

seen from the proceeding chapter, are connected with expenditures of vast amounts of labor and time. In as much as stellar statistics require an aquisitation of data on characteristics of as great a number of stars as possible, a project of unusual magnitude would be required for the spectrophotometric work. Also, the obtaining of the stars spectrum, to a certain degree limits the chance, for this particular instrument, to penetrate into the depth of the space, and thus limits the accumulation of the data, so sorely needed to interpret the problems of the constitution of the universe.

It is clear, that in order to avoid these difficulties, it is necessary to find characteristics of the distribution of the energy in the stellar spectra, that could be measured in the simplest way, and almost without any spectral dispersion of the stellar radiation. Such characteristics, connected with the color of the stars are known as the color equivalents. The measurement of the color equivalents is the basic problem of colorimetry.

In as much, as in the first approximation, the distribution of energy in the stellar spectra follows Planck's Equation and thus depends only upon a single parameter - the effective temperature, it becomes evident, that the color equivalents should be chosen in such a way, that they could be used to determine the effective temperature. For this it is sufficient that the color equivalent furnishes either the relative intensities of two parts of the spectrum of any width, or the wavelengths of the maximum intensities with certain methods of observation.

The parts of the spectrum may be cut out either by light filters, passing the radiation of certain part of the spectrum only, or by special apparatus, sensitive to different parts of the spectrum.

The color equivalents, chosen in one way or the other, should be of the same value, and, therefore interchangeable, but, in as much as once in a while, the stellar spectra exhibit deviations from Planck's law, and also due to the presence of the spectral lines, the color equivalents, computed for different parts of the spectra, are not always absolutely the same. Therefore, for the sake of the most complete characterization of the stellar radiation, it becomes interesting to determine as many various color equivalents as possible.

The computation of the effective temperature by the known color equivalent is quite difficult. In order to realize the nature of these difficulties, let us examine the case of a color equivalent that was computed as the ratio of the intensities of two parts of the spectrum, measured on a photographic plate.

Let $I(\lambda)$ indicate the function of the distribution of the energy in the spectrum of the star, and $F(\lambda, z)$ - the combined coefficient, expressing the transparency of the atmosphere at the Zenith distance z and the transparency of the optical system of the instrument. Then, the intensity of the radiation, that strikes the plate in every wavelength will equal $I(\lambda) \cdot F(\lambda, z)$. If we multiply this function by the sensitivity of the plate $V(\lambda)$, then the radiations of different wavelength will, in their photographic action, be practically reduced to the same wavelength. In the end, the integrated

intensity I, computed from the measurements of the photographic plate, will be expressed by the formula

$$I = \int_0^\infty I(\lambda) F(\lambda, z) V(\lambda) d\lambda, \qquad (14)$$

as it is natural to sum the intensities, reduced to the same wavelength.

Formula (14) written for two parts of the spectrum, allows the compution of T, if $I(\lambda)$ is computed by Planck's formula, and the functions $F(\lambda, z)$ and $V(\lambda)$ are already known.

The validity of formula (14) especially when it is applied to the wide parts of the spectrum, is open to many doubts. Really, as was elucidated in the 4, $V(\lambda)$ depends on the density of the image, and therefore, on the $I(\lambda)$ (the Purkinje Effect). Then, the rays of different wavelengths, even if they were reduced, from the point of view of the production of darkenings of some density, to the same wavelength, may be combine. We shall recall, that for example, the red rays, under certain conditions, may partially bleach the darkening of the plate. These objections remain in force for the visual measurements of integrated intensities.

Formula (14) is, undoubtedly correct for the observations with the bolometer, radiometer or the thermo-couple.

Theoretically, the refinement of Formula (14) for its application to the photographic and ocular measurements does not present any special difficulties. Then, by a special standardization of the plates, directed to the finding not only the function S(I) but of some additional important characteristics of the plate, it becomes possible to find very strict correlations between the effective temperature and the color equivalents.

However, such a problem is hardly of any practical value. It should be kept in mind, that the principal value of the color equivalents lies in the rapidity and ease of their determination. This demands, that all operations of the colorimetric observations be as simple as possible and problems such as determinations of relationship, or the correlation between the effective temperature and the color equivalents, or the correlations between two systems of color equivalents, are best studied empirically i.e. by the way of a direct comparison of all these quantities for the same stars.

All the color equivalents in use at present, can be divided in the following three groups: 1) The color equivalents, determined by a single measurement, as by a measurement of λ_{max}. These methods of study of the stellar spectra will be, from now on, referred to as the methods of effective wavelengths; 2) The color equivalents, computed from two measurements, i.e. by the determination of the ratio of the intensities of two parts of the spectrum. The methods of this type we shall call the Methods of Color Indices. To these two fundamental methods we should add 3) - The visual measurements of the stars. In as much as the eye distinguishes three basic colors - red, green and blue, actually, during such measurements we perform the determination of the comparative intensities of three parts of the spectrum.

§11 - The Effective Wavelengths.

That wavelength, eff, that corresponds to the intensity, that produces the greatest effect on the eye, or on the photographic plate, is called the effective wavelength. Thus λ_{eff} equals λ_{max} of the function I(λ) F(λ , z) V(λ).

The effective wavelengths were first mesaured by Hertzsprung photographically, and vusually by Comstock and Lay. The measurement eff is performed in the following way.

The wire grid, described in the 5, is placed in front of the objective. Then the observed will see the central image of the star, flanked by small spectra of different orders. The dispersion of these spectra is so small, that they look like, slightly oval, stars,

In the grids, commonly used, the width of the opening between the wires equals the thickness of the wires. For such grids (See §5), only the image of odd orders will be seen, and the brightness of the image of the first order will be less than that of the star, without the grid, by 2.5 magnitudes.

For the determination of the effective wavelength, a measurement is taken between the centers of gravity of two diffraction images of the first order. According to formula (8), this distance is determindd by the relationship.

$$l = \frac{2\lambda f}{s+d}$$

With l known, the λ , corresponding to it may be computed. This wavelenght is accepted as that λ_{eff}.

In as much as the stars image is never represented by a point, but has a definite diameter, the measured diffraction images present a superposition of many different spectra and, consequently, the eff, measured by this method may vary to a certain extent from that ideal effective wavelength that was defined at the beginning of this paragraph. But in as much as the value of the effective wavelength by its nature represents a certai color equivalent it is used, without any attention paid to tht variation.

It is obivious, that the effective wavelength depends on the quality of the objective, on the kind of plates used, and on the quality of the images Therefore, the effective wavelengths, determined by different instruments and at different locations, may differ systematically from eachother.

The connection between different systems may be foune empirically. In order to enable each observer to reduce his system to a certain standard system, Rosenberg Bergstrand made exact measurements of 25 curcynoakar stars.

The effective wavelenght depends, to a great degree, on thedensity of

the image and on the focusing. These corrections also can be found emperically, and usually, in the determination of the correlation between the $_{\mathrm{eff}}$ and the density of the image, the diameter of the central image is taken for the unit of density.

The correction of $_{\mathrm{eff}}$ for the atmospheric transparency, whenever z 45°, may be disregarded; evidently, when the introduction of this correction is desired, it may be found by the experimental method.

For demonstrating the changes of λ_{eff} with the color of the star, we include Table 6, containing the values of λ_{eff} for the various spectral types.

Table 6

Spectral Type	B_o	A_o	F_o	G_o	K_o	M
Photographic λ_{eff}	412	418	422	428	433	443
Visual λ_{eff}	565	565	570	570	575	579

The effective wavelengths, even if this can be measured with only rough approximation due to the condition just described, are very important, as the first reconnaissance, in the mass study of the stars. The $_{\mathrm{eff}}$ is a very convenient color equivalent which can be computed from a single measurement, and with a simple apparatus.

The high loss of light during the process of determination of the λ_{eff} is very aggravating; with the grids commonly used, the loss reaches 2.5 stellar magnitudes. In order to lessen this loss, Kluber has replaced the grid by an objective prism of a very small refracting angle (about 1°), that covered one half of the objective. Such a prism is placed above the center of the objective. The unobstructed parts of the objective produce a regular image of the star, while the prism produces a small spectrum, and the distance between the image of the star and the spectrum is measured.

The loss of light in this method is about 0.8 magnitudes and the accuracy of the measurement is about the same, as in the wire grid method.

Lindblad has proposed to replace the $_{\mathrm{eff}}$ by the $_{\mathrm{min}}$ and has defined it as the minimum distance between two diffraction spectra of the first order. However, the experiments have demonstrated, that the λ_{min} can be measured as accurately as the λ_{eff}.

At present, there is a vast number of effective wavelengths determined

The objectionable part of this kind of observation, is with the color filters, is the impossibility of avoiding the influence of the spectral lines and the moliculer bands.

The color filters are poorly adapted to the determinations of the color equivalents which are interesting from the point of view of the mass study of the stellar spectra, as they cut out too narrow parts of the spectrum. For such measurements it is necessary to widen greatly the studied parts of the spectrum by the use of very "wide" filters, or by the difference of the spectral sensitivities of the employed apparatus.

Let m_1 be the stellar magnitude of a certain star when observed with a certain apparatus of a spectral sensitivity $V_1(\lambda)$, and m_2 be the stellar magnitude of the same star, observed with an apparatus of a spectral sensitivity $V_2(\lambda)$. Let us assume, further, that m_1 corresponds to shorter wavelengths, than m_2. Then the difference of the stellar magnitudes $C.I.=m_1 - m_2$, measured with an exactitude of a certain constant, common to all stars is called the Color Index of the star. By the use of the formula (14), the color index can be found thus:

$$C.I = -2.5 \lg \frac{\int_0^\infty I(\lambda) F_1(\lambda, z_1) V_1(\lambda) d\lambda}{\int_0^\infty I(\lambda) F_2(\lambda, z_2) V_2(\lambda) d\lambda} + C. \quad (15)$$

The constant C in this formula, dependent on the arbitrary choice of the zeropoint of the two scales of the stellar magnitudes is found from the condition, that $m_1 = m_2$ for the stars of the type A_0. This is an entirely arbitrary condition, that is very convienient from the practical point of view, and can be replaced by another, that has a definite physical significance.

The certain stars, with a known distribution of energy in the spectrum - $I(\lambda)$ it is possible to compute the values of the integrals in formula (15); then, from a comparison of the observed color indices with the right half of this formula, the value of the quantity C can be computed.

The construction of such a system of the absolute color indices, introduced by Brillo, is not very important for the majority of astrophysical problems, in as much as the object of the introduction of the Color Indices is the obtaining of the characteristics of the color of the stars, that can be performed, with a sufficient accuracy to a desired limit of the error.

Formula (15) contains, under the signs of integration, the functions that depend on the Zenith distance of the star. In order to obtain uniformity of the system of Color Indices, the stellar magnitudes m_1 and m_2 are referred to the Zenith, by an assumption, that in this formula

$$z_1 = z_2 = 0.$$

The stellar magnitudes, measured by any method of observation, may be referred to the Zenith in the same way, as is done in the case of the visual magnitudes. The details of this operation were given in Chapter III, and here we will recall only that formula (2) is usually accepted as valid for

the integrated radiation of the star. In this case P is accepted as a certain effective coefficient of transparency. Owing to the comparatively low accuracy, that is required from the determinations of stellar magnitudes, this method, not correct mathematically, produces satisfactory results.

In precise observations it is necessary to avoid the observations at 2755° or 60°. A refinement of the determination of the Color Indices can be effected by a simultaneous determination of m_1 and m_2, i.e. at the same z, and identical atmospheric conditions. Evidently (see formula 15) the color index determined by this method still may differ to a certain degree from the color index at $z_1 = z_2 = 0$.

Every apparatus used for these determinations, creates its own system of Color Indices. If this function differ little (as the systems of color indices, based on the comparison of the visual and photographic stellar magnitudes, but obtained by different observers and on different instruments), such systems are combined into a single system.

This combining of the systems is done empirically. By a comparison of the Color Indices of the stars common to both systems, a definite correlation between the systems is established. This correlation may be presented analytically, and usually it is possible to limit oneself to linear terms, and only in some cases does it become necessary to introduce the terms of the second degree.

In those cases, when in the different systems the function $V(\lambda)$ and $F(\lambda, c)$ differ greatly, no combining is done, and the resultant systems are regarded as distant types of the color indices. A study of such detached types of color indices is important, for example, for the understanding of the deviations of the stellar radiation from Planck's formula.

Let us review these most important types of color indices.

1 - The photographic color index is defined as the difference of the photographic and visual stellar magnitudes, or as $m_{ph} - m_v$. For the determination of the photographic stellar magnitudes, the common, not specially sensitized, plates are used. As is evident from the tables, included in 3 and 4, the curves of sensitivity of the plates and of the eye coincide only very little, and therefore, the photographic color index denotes the relative intensity of two separate, almost not coinciding, part of the spectrum.

The photographic color indices were first computed by Schwarztschild from a comparison of the photographic magnitudes of the Gottinger Catalogue with the visual magnitudes of the Potsdam Catalogue (The Gottinger Actinometry) Later, King computed the color indices of 200 bright stars from a comparison of his own numerous measurements of the photographic magnitudes with the visual magnitudes measured at Harvard. The King System is accepted now as the basic system of photographic color indices.

In as much as photographic photometry has a number of very important

advantages as computed with visual photometry, it becomes desirable to obtain the visual stellar magnitudes also with the aid of the photographic plate. This can be achieved by using a yellow filter during an exposure on an othochromatic plate. Such system of the stellar magnitudes, that may differ somewhat from the visual, is called the System of Photo-visual Stellar Magnitudes (m_{pv}). The difference m_{ph} - m_{pv} is called the Photo-visual Color Index.

The system of the photo-visual color indices of the Yerkes Observatory (Parkhurst) is accepted as the basic system. The Yerkes photometric studies include 670 stars near the pole. The resultant curve of the transparency of the yellow filter and of the sensitivity of the plate used in this system, is very close to the curve of the sensitivity of the eye; the maximum of this curve coincides exactly with the $_{max}$ of the eye.

2 - The Photoelectric Color Indices are computed from the measurements of the stars with the photo-electric apparatus. In as much as the photo-ele electric units, using different metals, have different curves of sensitivity, theoretically it is possible to compute the color indices from a comparison of the stellar magnitudes, as measured by different photo-electric units. In practice, this was proved to be not convenient. It is much simpler to make the measurements on the same apparatus, but through different filters.

Such measurements of a large number of stars (459) were performed by Guthnick and Bottlinger at the Babelsberg Observatory. The stellar magnitudes were measured through two filters: yellow ($_{max}$= 457μμ) and blue ($_{max}$= 437μμ). Therefore, the measured intensities were of, relatively, near parts of the spectrum. The difference of the magnitudes m_{blue} - m_{yellow} was taken to be the photo-electric color index. This color index is ofter called The Bottlinger Color Index.

The photo-electric color indices, owing to the high precision of the photo-electric measurements, present very fine color equivalents.

3 - The Thermal Color Index is defined, as the "difference between the visual and bolometric stellar magnitudes." The bolometric magnitude is meant to be the energy intensity of the star, measured by an apparatus the sensitivity of which is the same for all wavelengths i.e. radiometer, bolometer and thermo-couple.

Nicholson and Pettit have measured a great number of bolometric magnitudes, with the 100" reflector, equiped with a highly sensitive thermo-element, which allowed the stars to the 5th magnitude to be measured. The thermol indices were computed from a comparison with the Harvard visual magnitudes.

Besides, in order to obtain a better idea of the intensity of the infra-red radiation of the stars, the measurements were taken with and without interposition of a water screen in front of the thermo- element. The water screen acted as a filter that entirely absorbed the rays with a 1300μμ, but passed the visual and the near ultra violet radiation.

Table 7 lists the color indices of the three basic types, relative to the spectral class of the stars.

In §7 it was pointed out that there is no strict, simple correlation between the spectral types and the effective temperatures. It is evident that the same should be true for the color indices. The data of this table should be regarded, not as a relationship between the spectral types and the color indices, but more as a correlation between the effective temperatures, corresponding to those types, as shown in the Table 4, and the color indices.

Table 7

Spectral Type	Photographic Color Index	Photo-electric Color Index	Thermal Index
B0	- 0.33	- 0.09	- 0.1
A0	0.00	0.00	0.0
F0	+ 0.33	+ 0.12	+ 0.3
G0	+ 0.67	+ 0.33	+ 0.7
K0	+ 1.12	+ 0.53	+ 1.2
M0	+ 1.73	+ 0.84	+ 2.3

If we shall attempt to compute the temperature by the color indices, according to the experimentally established correlations, then in some cases, different color indices may produce different temperatures. The cause of this is in the possible deviation from Planck's formula.

The computations of the effective temperatures by the color indices, can be performed also theoretically, by the use of formula (15).

With the aid of formula (14) it is possible to compute for the given system of stellar magnitudes, that wavelength, which should be ascribed to the observed magnitude.

This wavelength, for which the difference of the intensities of the monochromatic radiations of different stars, equals the difference of the integrated magnitudes of these stars, was called by Brille the iso-photic wavelength. It is evident, that the iso-photic wavelength is not necessarily the same as the eff. Within the same system it may vary, depending on the spectral type of the star.

The color indices, found from a comparison of two systems of stellar

magnitudes, are determined by the observations of the same star that were performed, as ofter happens, over a large period of time. This condition may affect the accuracy of the computations of the color indices, owing to the possible changes of the atmospheric transparency, or to the possible variations of the brightness of the star itself.

The Bottlinger Color Indices are free from these errors, as they were computed from measurements that followed each other by a few minutes. By the use of specially constructed astro-graphs, a practically simultaneous exposure of the plates is achieved for the determinations of the photo-visual color indices.

The photo-visual color indices can be easily obtained in short intervals of time by the "exposure ratio" method proposed by Sears. Several photographs of the star are made on a orthochromatic plate, without any filters, with the exposures diminishing in geometric progression. Also, one exposure is made through the yellow filter.

By a measurement of the image of the star, obtained through the filter, in the terms of the scale of different exposures, it is possible to find the ratio of the exposures for the images obtained with and without the filter. Such a ratio may be used itself as a certain color equivalent. It is also evident, that by applying the Schwarztschild Law, it is easy to establish the correlation between the "exposure ratio" and the color index.

The Sears Method, in the form we have described, is not very accurate as the Scwarztschild's Index changes with the wavelength, and due to possible deviations from the Schwarztschild's Law. Sears used a number of contrivances to avoid these errors, but we shall omit their discription.

The simultaneity in the determinations of the color indices, is one of the important advantages of the Tikhoff Method of the measurement of the color indices by the, so called "longitudinal spectrograph."

The longitudinal spectrograph is a common refractor, with the central part of the objective covered by a diaphragm. An orthochromatic plate is placed at the focus of the visual rays of this objective.

The visual ray form an ordinary image of the star, while the rays of the short wavelengths, form rings, around this central image, the diameter of these rings increasing with the decrease in wavelength. Owing to the lowered sensitivity of the plate to green light the photograph will show a central image, surrounded by a ring at a certain distance from it. This ring comes out more distenctly for the white stars, than for the red, and consequently, the general appearance of the photograph gives a certain idea about the color of the star. By a measurement of the darkenings of the central image and of the ring it is possible to compute quantitative characteristics of the color of the star. Evidently, these characteristics can be correlated with the color indices.

The longitudinal spectrograph method can hardly be expected to find an extensive application in the studies of faint stars, owing to a large loss in the brightness of the stars, due to the obstruction of the objective by the diaphragm, and to the violet images being out of focus. A strict photometric interpretation of the photographs, produced by the longitudinal spectrograph is also difficult.

§13 - The Visual Determinations of the Color of the Stars.

The determinations of the apparent color of the stars embraces an extensive set of data, and goes back to the very first astronomical observations. The catalogues of these colors were composed long before an idea of a correlation between the color of the stars and their temperatures could appear.

Any color can be produced by mixing the three basic colors: red, green and blue, which is a fact proven by experiments. As an explanation of the color perception, that we experience. Young proposed the following theory that was later developed by Helmholtz.

Our color impressions are due to the exciting of three different photosensitive elements, present in the retina. The exciting of one type of these photo-sensitive elements creates an impression of red color, the exciting of another - the impression of gree, and the exciting of the third - of blue.

Each of these photo-sensitive elements reacts with a different force to the radiations of different wavelengths, i.e. each photo-sensitive element has a sensitivity curve of its own. Thus, for instance, the violet end of the spectrum excites the impressions of red and blue color, while a wavelength of 500μμ excites the impressions of all three basic colors.

If a certain radiation produced an equal excitation of all photo-sensitive bodies, it produces an impression of a white color. Daylight is accepted as white light, i.e. it is presumed to cause equal excitation of all photosensitive elements. This assumption allows a common scale to be established for the three curves of sensitivity.

All the colors that we are able to preceive, can be represented graphically, or numerically, with the aid of the so-called Helmholtz's Triangle.

Certain three masses, proportional to the excitation of the red, green and blue bodies, under the influence of a certain radiation, assumed to be placed at the apex, of a common equilateral triangle. The center of gravity of this system defines the resultant color impression. The center of the will be the c lors nearly white, or as it is said, the colors of a low intensity.

Let us take a certain definite radiation, with the known I(λ). By the use of the curves of sensitivity for the different photosensitive elements, it is possible, by an integration, to compute the resultant excitation of the

red, green and blue elements, that we shall call r, g and b, respectively.

The color, corresponding to this radiation, will be expressed by the center of gravity of the points with masses r, g and b. If we assume the side of the triangle/equal to 1, the apex with the mass r as the zero point of the coordinates, and the straight line, connecting the points r and b as the axis of absoissa, then the resultant color impression will correspond to the following quantities:

$$X \frac{b+\frac{1}{2}g}{r+b+g}; \quad Y = \frac{\frac{\sqrt{3}}{2}g}{r+b+g}, \qquad (16)$$

that are the coordinates of the center of gravity of the Helmholtz's Triangle.

By the measurements of the colors of the different radiations it is possible to establish the correlation between the subjective measurements of the colors (the names of the color), and the quantities X and Y. Assuming, that the stars radiate according to Planck's Law, and introducing a correction for the selective absorption of the light by the terrestrial atmosphere, Bottlinger has computed the values X and Y for stars of different effective temperatures, from 2000 to . He has found, that the colors of the stars, can be only of hues: yellow and bluish. The stars with a temperature around 2000° should produce an intense yellow light. With the increase of temperature, the yellow color remains, but its intensity diminishes. At 6000°, the stars should produce a white light, and at 20000° - a blue of the intensity of the blue sky in the mountains.

If we examine the observed colors of the stars, we shall find, that the observers define the color of the stars with a temperature of 2000°, as red, the color of the stars with a temperature of 6000°, as yellow, and the stars of higher temperatures as white. The blue color is hardly ever noted, with the exception of a few binaries.

The reason of this shift of the observed color scale related to the computed one, lies probably in some peculiarities of the perception of the star light. First - the eye appraises the color of the point like objects, as the stars, only with difficulty. Then, during the observations at night, the eye is under different conditions, than in the daytime, and finally, with the exception of close double stars, there are no objects for comparison, and thus the eye is unable to appraise the color by contrast.

We know, that theoretically, it is very difficult to establish a correlation between the effective temperatures and the colors of the stars. However, such a correlation can be easily found, experimentally, for every scale of colors.

The most widely used scale of colors is that of Osthof. In this scale all colors are divided into 10 classes, with 0 corresponding to the pure white, and 9 to red.

The old catalogues, such as the Potsdam Durchmusterung of 1897 contain the visual appraisals of the colors of the stars. At present with the exception of some extra ordinary cases, there are no reasons to make such appraisals, owing their extreme crudeness (these determinations of color depend greatly

upon the brightness of the stars)

Instead of making subjective evaluations of the colors of the stars, we can measure these colors by a comparison of the color of the star with the color of some source of reference. The apparatus used for this purpose is called the colorimeter.

The colorimeter is an ordinary visual photometer, equipped with an additional, specially selected, wedge shaped light filter. By changing the position of this wedge, it is possible to equalize the color of the star and the color of the artificial source of comparison.

In as much as the difference of colors is well appraised only at equal brightness and for objects of a certain (minimum) area, the common photometric appliances, that allow the equalization of the brilliancies, without any changes of color, as the photometric wedge and the nicol prism, are used in the colormetric technique.. Also, the stars are observed somewhat out of focus.

The wedge filters are used in two colors: red and blue. If the wedge filter is placed before the stars it should be, evidently, red and if placed in front of the artificial reference light, it should be blue. The first method is not as convenient as the second, due to a weakening effect on the brightness of the star.

Let us see now, how the colorimetric observations allow the determination of the distribution of the energy in the spectrum of the star. Let us assume, that the distribution of energy depends only on one parameter - the effective temperature T. Then the problem will consist of the computation of the quantity T by two readings, taken during the colorimetric observations (the reading for the colored wedge, and the nicol prism reading). Let us assume, that the observations are made with the red wedge, and introduce the following signs: $I(\lambda, T)$ - the brilliancy of the star in every wavelength. $I'(\lambda, T')$ - the brilliancy of the reference light, K - reading of a red filter, (λ) - a certain function, proportional to the coefficient of absorption of that filter and Q - the factor, that expresses the diminution of the brilliancy of the artificial light, derived from the readings of the photometer. Then, we assume, according to 2, the absorption of the terrestrial atmosphere to be $k(\lambda)$ and the mass of air, along the path of the ray from the star with a zenith distance z, to be $M(z)$.

We can produce the equations of our problem if, first, we write, according to formula (14) and equation between the observed brightness of the star and the brightness of the reference light, and second, if we equate, according to formula (16) the coordinates X and Y of the color of the star and the coordinates of the color of the reference light in the Helmholtz's Triangle. Evidently we shall satisfy this system of equations if, following Wilsing, we assume, that, the equation

$$I(\lambda,T)e^{-k(\lambda)M(z)}e^{-K\phi(\lambda)} = QI'(\lambda,T') \qquad (17)$$

is valid for every wavelength. In other words, we make an assumption, that if the colors of the objects and their integrated brightnesses coincide, their intensities, in every , will also coincide. It is evident, that Equation (17) may be valid, when the functions, entering it, are not arbitrary. Wilsing has demonstrated, that this equation may be satisfied, if we use such functions as:

$$
\begin{aligned}
I(\lambda, T) &= C_1 \lambda^{-5} e^{-\frac{C_2}{\lambda T} + \gamma_0 + \frac{\gamma_1}{\lambda}}, \quad \text{exponent} \\
I'(\lambda, T) &= C_1 \lambda^{-5} e^{-\frac{C_2}{\lambda T'} + \gamma_0' + \frac{\gamma_1'}{\lambda}} \quad \text{exponent} \\
k(\lambda) &= k_0 + \frac{k_1}{\lambda}, \\
\Phi(\lambda) &= B_0 + \frac{B_1}{\lambda}.
\end{aligned}
\tag{18}
$$

Here an assumption was made, that the radiations of the star and of the reference light follow the law of the radiation of a black body, and in Planck's Formula in expansion of $\lg(1-e^{-\frac{C_2}{\lambda T}})$ in powers of $\frac{1}{\lambda}$, two terms were taken; γ_0 and γ_1 are functions of T only. Almost for all stars, the error of such a expansion does not exceed 1°/o. Almost the same accuracy is obtained by the substitution of $k(\lambda)$ by the expression in formula (18).

The wedge must be selected in such a manner, as to correspond, as closely as possible to the condition $\Phi(\lambda) = B_0 + \frac{B_1}{\lambda}$. By substitution of the functions (18) in the formula (17), we see, that the resultant equation can be valid for all wavelengths, if

$$\frac{-C_2}{T} + \gamma_1 - k\,M(z) - B_1 K_1 = -\frac{C_2}{T'} + \gamma_1 \tag{19}$$

and

$$C_1 e^{\gamma_0} - k_0 M(z) - B_2 K_0 = C_1' \, Q e^{\gamma_0'}. \tag{20}$$

We can compute T from Equation (19). The values of the functions γ_0 and γ_1 can be tabulated for different T.

The formula (20) serves for the determinations of the brightness of the star. From this formula the ratio $\frac{C_1}{C_1'}$ can be computed; then, by formula (14) with T and T' known, it is not difficult to find the visual brightness of the star.

In practice, during the observations with a colorimeter, by limiting the measurements to relative ones, there is no necessity of measuring the brightness of the reference light and its effective temperature.

Really, the observations of the studied star and of a certain reference star with a known effective temperature allow two systems of the equations of the type of (19) and (20) to be written. By subtracting one of the equations of the type of (19) from the other, we have a correlation between the effective temperatures of these two stars. If we disregard the small quantities and the resultant equation can be written thus:

$$\frac{C_2}{T} - \frac{C_2}{T_0} = B_1 (K_0 - K_1) + k_1 \left[M(z_0) - M(z) \right] \tag{21}$$

where K_0 is the reading of the color wedge for the reference star and z_0 is the zenith distance of that star.

The colorimeter with a blue wedge was constructed by Fessenkoff. The wedge was selected in such a way, that it was possible to change the color of the reference light from reddish -yellow to pure white. It is evident, that the theory of the colorimetric measurements with a blue wedge, can be developed anologously to the theory of the Wilsing colorimeter. With that colorimeter, Fessenkoff has made a large number of measurements of the stars down to the fourth magnitude.

The advantage of the determinations of the effective temperatures with the colorimeter lies in the extreme simplicity of the interpretation of the observations (see Formula (21)). The disadvantages of this method are: 1) the difficulty of the comparison of the colors; for some stellar type this error is very significant, and 2)the impossiblity of measurements of the temperatures of the faint stars, that present, really, the greatest interest.

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